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On nonautonomous Markov evolutions in continuum

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Abstract. The nonautonomous Cauchy problem in a scale of Banach spaces is investigated. The existence and uniqueness of solutions to this problem is proven. The obtained results are applied to several dynamics of Markov evolutions in continuum (e.g. spatial logistic model, Glauber dynamics, etc.).

Contents

1 Introduction

A possible way of describing dynamics of complex systems of interacting particle is to assume that the elementary acts of the evolution occur at random and the evolution itself is Markovian. Among the mentioned elementary acts one can distinguish birth, death and motion. The rates at which they occur may

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depend on the actual state of the system and on the environment. Among various problems coming from the natural and life sciences the existence of state evolutions for wide classes of intensities (e.g. time dependent intensities) seems to be one of the fundamental problem. The evolution of states is informally given as a solution to the initial value problem:

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle F, \mu_t\rangle = \langle LF, \mu_t\rangle, \quad \mu_t|_{t=0} = \mu_0,$$

provided, of course, that a solution exists. Here L is an informal generator which describes the functional evolution of the system

$$\frac{\partial}{\partial t}F_t = LF_t, \quad F_t|_{t=0} = F_0$$

and

$$\langle F, \mu\rangle := \int_\Gamma F(\gamma)\mathrm{d}\mu(\gamma).$$

One of the aims of the present paper is to develop methods to solve nonautonomous Cauchy problems in a scale of Banach spaces $\mathbb{B}_\alpha$, which will be used to treat systems with time or enviroment dependent intensities. Our main technical tool is a general theorem by M. Safonov from [26] and several conclusions, obtained in the present paper. Using this theorem we will prove the existence of solutions on a bounded time interval for several models and in some cases we will give conditions for the existence of solutions on unbounded time intervals. The first part will be devoted to the general theory of nonautonomous Cauchy problems on scales of Banach spaces. A version of the general theorem by Safonov for linear operators will be proven. Afterwards we will extend this theorem for weaker assumptions, where the generator consists of two parts $L = A + B$ and only the second part satisfies the assumptions of the general theorem of Safonov. This technique will be used to prove a continuous dependence of the solutions on parameters. Markov evolutions of continuous interacting particle systems were studied by many authors for time independent coefficients. In the present paper we are going to be focused on nonautonomous models of birth and death type. However, the abstract results obtained in this paper may be applied also to other classes of Markov evolution. In our approach, populations will appear as particle configurations forming the following phase spaces

$$\Gamma = \Gamma(\mathbb{R}^d) = \{\gamma \subset \mathbb{R}^d : \ |\gamma \cap K| < \infty, \ \ \forall K \subset \mathbb{R}^d \text{ compact}\}.$$

One of the most simplest models of birth and death type is the so-called Sourgailis model. The mechanism of its evolution is given by the following heuristic generator

$$(LF)(\gamma) = m\sum_{x\in\gamma}\left(F(\gamma\backslash x) - F(\gamma)\right) + \kappa\int_{\mathbb{R}^d}\left(F(\gamma\cup x) - F(\gamma)\right)\mathrm{d}x \qquad (1)$$

with $m, \kappa > 0$, cf. [27, 28]. In (1), the first term describes the death of the particle located at $x \in \gamma$ occurring independently with the rate $m > 0$. The second term in (1) describes the birth of a particle at $x \in \mathbb{R}^d$ with the constant rate $\kappa > 0$, which is independent of $\gamma \in \Gamma$. The corresponding state evolution as well as ergodic properties of the process were recently studied in [3].

Another model for Markov evolution which includes interaction between particles in the birth mechanism is, for example, the continuous contact model. It can be described by the formal Markov generator

$$(LF)(\gamma) = m \sum_{x \in \gamma} (F(\gamma \backslash x) - F(\gamma)) + \int_{\mathbb{R}^d} \sum_{x \in \gamma} a(x,y)\,(F(\gamma \cup y) - F(\gamma))\,\mathrm{d}y,$$

where $m > 0$ and $a(x,y) > 0$. The first term (death) is the same as for Sourgailis model and the second term describes the birth of a new particle at $y \in \mathbb{R}^d$ given by the whole configuration γ with the rate $\sum_{x \in \gamma} a(x,y) > 0$. This model was studied in the translation invariant case, i.e. $a(x,y) \equiv a(x-y) = a(y-x)$, in [18] and [20]. In [20] the authors proved the existence of the corresponding process for a dispersion kernel $a \in L^p(\mathbb{R}^d)$, $p > 1$ with compact support. The evolution of correlation functions and invariant states for the contact model were studied in [18].

A generalization of the previous model which includes local regulation in death is described by

$$\begin{aligned}(LF)(\gamma) &= \sum_{x \in \gamma} \left(m + \sum_{y \in \gamma \backslash x} a^-(x,y) \right) (F(\gamma \backslash y) - F(\gamma)) \\ &\quad + \sum_{x \in \gamma} \int_{\mathbb{R}^d} a^+(x,y)(F(\gamma \cup y) - F(\gamma))\mathrm{d}y\end{aligned}$$

with a competition kernel $a^- : \mathbb{R}^d \times \mathbb{R}^d \longrightarrow \mathbb{R}_+$, a dispersion kernel $a^+ : \mathbb{R}^d \times \mathbb{R}^d \longrightarrow \mathbb{R}_+$ and a mortality rate $m > 0$. Such model is called spatial logistic model or Bolker-Dieckman-Law-Pacala (short BDLP) model. A detailed analysis of this model in the case of translation invariant kernels may be found in [5, 7].

Another example of birth and death type dynamics is a non-equilibrium Glauber-type dynamics, described by

$$(LF)(\gamma) = m \sum_{x \in \gamma} (F(\gamma \backslash x) - F(\gamma)) + z \int_{\mathbb{R}^d} e^{-E(x,\gamma)} (F(\gamma \cup x) - F(\gamma))\mathrm{d}x$$

with $E(x,\gamma) = \sum_{y \in \gamma} \phi(x,y)$, where $\phi : \mathbb{R}^d \times \mathbb{R}^d \longrightarrow \mathbb{R}$ is a pair potential. For non-negative translation invariant potentials this model was discussed in [4, 6, 11, 12, 17, 19]. The reversible states for these dynamics are grand canonical Gibbs measures. This fact gives a standard way to construct properly associated stationary Markov processes using the corresponding (non-local) Dirichlet forms related to the considered Markov generators and Gibbs measures. These processes describe the equilibrium Glauber dynamics which preserve the initial Gibbs state in the time evolution, see e.g. [19]. The construction of a non-equilibrium Glauber-type dynamics was done in [17]. It was based on a general approach for the construction of non-equilibrium evolutions developed in [16]. In [6] the authors have shown that the correlation functions corresponding to Glauber dynamics converge to the correlation functions of the equilibrium state. Using Ovsjannikov-type technique in [4] an evolution in a scale of Banach spaces for quasi-observables and correlation functions was proved. In contrast to [6] in the present paper no conditions on z and $\beta = \int\limits_{\mathbb{R}^d} 1 - e^{-\phi(x)}\mathrm{d}x$ are imposed. The same technique was used in [11] to analyze the evolution of Bogoliubov generating functionals. In the present paper the similar arguments will be used to generalize the existence results, although only existence and no further properties will be studied.

Chapter 3.5 of this paper is devoted to the general birth and death Markov dynamics, given by

$$(LF)(\gamma) = \sum_{x\in\gamma} d(x,\gamma\backslash x)(F(\gamma\backslash x) - F(\gamma)) + \int\limits_{\mathbb{R}^d} b(x,\gamma)(F(\gamma\cup x) - F(\gamma))\mathrm{d}x. \quad (2)$$

Using a semigroup approach the existence of a solution to the corresponding Cauchy problem for quasi-observables and correlation functions were proven, cf. [8]. The authors further have shown that under several conditions there exists a unique solution to the stationary equation $L^{\Delta}k = 0$, which can be constructed by the "generalized Kirkwood-Salzburg" operator. Here L^{Δ} denotes the generator for the evolution of correlation functions. In this paper we will also study these equations in the class of sub-Poissonian correlation functions.

The structure of the paper can be described as follows. At the beginning we give a brief outline on the continuous Sourgailis model. An explicit solution for correlation functions k_t will be given and differentiability on some Banach spaces will be proven, assuming the initial data are regular enough. The possibility to solve all equations explicitly suggests this model as a play model. Further questions concerning this model deal with random time dependent coefficients.

In sections 3 and 4 the existence of solutions for quasi-observables in the case of BDLP and Glauber dynamics will be proven and, further, the evolution of correlation functions and Bogoliubov generating functionals be considered for Glauber dynamics. The assumptions are likely the same as for the time independent results, despite all inequalities should hold uniformly in time.

In the last section we will prove existence of solutions for infinite time intervals for general birth and death dynamics with the time dependent coefficients. Here the time dependence will enter only multiplicatively, i.e. $d_t = m(t)d$ and $b_t = \kappa(t)b$ (cf. (2)), since we need precise information about the domains of the corresponding generators.

2 Evolution Equations in Scales of Banach Spaces

2.1 General setting

Let X be a Banach space, let $[0,T] = I \subset \mathbb{R}$ be a compact interval, and let $(L(t), D(L(t)))_{t\in[0,T]}$ be a family of operators on X. Our main object of investigation is the following nonautonomous Cauchy problem

$$\frac{\partial u}{\partial t}(t) = L(t)u(t), \quad t \geq s, \; t \in I, \quad u(s) = u_0 \tag{3}$$

on X for $0 \leq s < T$. Such equations were analyzed in, e.g. [13, 22, 23]. The aim is to construct an evolution family

$$\Delta \ni (t,s) \longmapsto U(t,s) \in L(X),$$

where $\Delta = \{(t,s) \in I \times I : \quad s \leq t \}$. This map should be strongly continuous and should have, instead of the usual semigroup property, the evolution family property

$$U(s,s) = id_X, \quad U(t,q)U(q,s) = U(t,s), \quad 0 \leq s \leq q \leq t \leq T.$$

In order to give sense to the right hand side of (3), i.e. $L(t)u(t)$, we should assume $u(t) \in D(L(t))$ or more generally

$$u(t) \in \bigcap_{s\in[0,T]} D(L(s)) \subset X.$$

In general it is difficult to characterize the explicit structure of $D(L(t))$, which is one of the major difficulties in this approach. Therefore one restricts to some smaller subspace. Assume there exists a Banach space $Y \subset \bigcap_{t\in I} \mathrm{Dom}(L(t)) \subset X$ such that for each $u \in Y$ the mapping

$$\Delta \ni (t,s) \longmapsto U(t,s)u \in X$$

is differentiable with derivatives

$$\frac{\partial U}{\partial t}(t,s)u = L(t)U(t,s)u, \quad \frac{\partial U}{\partial s}(t,s)u = -U(t,s)L(s)u.$$

Then we can formally write the solution to (3) as

$$u(t;s,u_0) = U(t,s)u_0.$$

Similarly, the expression $L(t)U(t,s)u_0$ would be well-defined if we assume $U(t,s)u_0 \in Y$, so $U(t,s)Y \subset Y$, which will be assumption in Theorem 2.3. This considerations motivate the following definition of a solution to the above nonautonomous Cauchy problem (3), which can be found, e.g., in [24].

Definition 2.1. Let X, Y be Banach spaces such that $Y \subset X$ is continuously and densely embedded. For a family of operators $(L(t), D(L(t)))_{t\in[0,T]}$ assume

$$Y \subset \bigcap_{t\in[0,T]} D(L(t)) \subset X.$$

A function $u = u(t)$ is called Y-valued solution of the nonautonomous Cauchy problem (3) with initial condition $u_0 \in Y$, if it has the following properties

1. $u \in C([0,T]; Y) \cap C^1([0,T]; X)$
2. u solves (3).

The derivatives at $t = 0$ and $t = T$ will be always defined by

$$\frac{\partial u}{\partial t}(0) = \lim_{h\to 0, h>0} \frac{u(h) - u(0)}{h}, \quad \frac{\partial u}{\partial t}(T) = \lim_{h\to 0,\quad h>0} \frac{u(T) - u(T-h)}{h}.$$

Note that in contrast to a classical solution we impose continuity in the Y-norm, which is a stronger condition than just $u(t) \in Y \subset D(L(t))$. Contrary to the general semigroup theory, where the semigroup is always differentiable on the domain of its generator, it is possible that an evolution family is nowhere differentiable. Now we will state two results due to [24] for existence of evolution families under conditions know as “the hyperbolic case”. For this let us recall the definition of admissibility.

Definition 2.2. Let $(L, D(L))$ be the generator of a C_0-semigroup $(T(t))_{t\geq 0}$ on X and $Y \subset X$ a subspace. Y is said to be L-admissible if $T(t)Y \subset Y$ holds and the restriction $T(t)|_Y$ is a C_0-semigroup on Y.

In [24] it was shown that this is equivalent to the condition that the part $\widetilde{L}$ of L on Y is again a generator of a C_0-semigroup. This semigroup is then given by restricting $T(t)$ to Y. The part $\widetilde{L}$ of L on Y is defined as

$$D(\widetilde{L}) = \{u \in Y \cap D(L) : \quad Lu \in Y\}, \quad \widetilde{L}u = Lu, \quad \text{for } u \in D(\widetilde{L}).$$

Theorem 2.1 ([24]). *Let X, Y be Banach spaces such that Y can be densely embedded in X and let $(L(t), D(L(t)))_{t\in[0,T]}$ be generators of C_0-semigroups $((e^{\tau L(t)})_{\tau\geq 0})_{t\in[0,T]}$ on X. Assume that the following conditions are satisfied:*

1. *$L(t)$ is Kato-stable, i.e. $\exists M \geq 1$ and $\omega \in \mathbb{R}$ such that $(\omega, \infty) \subset \rho(L(t))$ for all $t \in [0,T]$ and*

$$\|e^{\tau_k L(t_k)} \cdots e^{\tau_1 L(t_1)}\|_X \leq M e^{\omega \sum\limits_{j=1}^{k} \tau_j}$$

for all $0 \leq t_1 \leq \dots \leq t_k \leq T$, $k \in \mathbb{N}$ and $\tau_1, \dots, \tau_k \geq 0$, where $\rho(L(t))$ denotes the resolvent set of $L(t)$.

2. *$Y \subset \bigcap\limits_{t\in[0,T]} D(L(t))$ and*

$$I \ni t \longmapsto L(t) \in L(Y, X)$$

is continuous in the uniform operator topology.

3. Y is $L(t)$-admissible for all $t \in [0,T]$ and $\widetilde{L}(t)$ as the part of $L(t)$ in Y is Kato-stable.

Then there exists a unique evolution family $(U(t,s))_{(t,s)\in\Delta}$ with the properties:

1. $\|U(t,s)\|_{L(X)} \le Me^{\omega(t-s)}, \quad (t,s) \in \Delta$

2. $\left(\frac{\partial U}{\partial t}\right)^{+}(t,s)|_{t=s}u = L(s)u$

3. $\frac{\partial U}{\partial s}(t,s)u = -U(t,s)L(s)u$

for $u \in Y$. Here the derivatives are considered in the sense of the norm in X and $\left(\frac{\partial U}{\partial t}\right)^{+}(t,s)|_{t=s}u$ is the right-sided derivative evaluated at (s,s).

Remark 2.1.

1. Kato-stability is neither necessary nor a sufficient condition for the existence of an evolution family. In [23] the authors gave a counterexample, where an evolution family exists, so the Cauchy problem (3) is well-posed, but the stability condition is not satisfied.

2. The main idea of the proof is to consider a sequence of with respect to t piecewise constant operators $A_n(t)$ and define appropriate evolution families $U_n(t,s)$, which are piecewise continuously differentiable on X for $u \in Y$. After showing the existence of a limit $U(t,s)$ in the strong sense on $L(X)$ it remains to show that the differentiability property still holds.

3. It is possible to replace continuity of $t \longmapsto L(t) \in L(Y,X)$ by the weaker assumption
$$L(\cdot) \in L^1([0,T], L(Y,X)).$$
In this case the strong differentiability for $(t,s) \in \Delta$ holds only almost everywhere.

To obtain stronger differentiability properties for $U(t,s)$ we should know further properties of the evolution family. In a scale of Banach spaces these properties can be easily checked. As already mentioned we should assume $U(t,s)u \in Y$ for $u \in Y$ to give meaning to the expression $\frac{\partial U(t,s)}{\partial t}u = L(t)U(t,s)u$. This will be the content of the next theorem, cf. [24].

Theorem 2.2. *Let $X, Y, L(t), U(t,s)$ be as in Theorem 2.1. If $U(t,s)Y \subset Y$ holds and the mapping*
$$\Delta \ni (t,s) \longmapsto U(t,s)u$$
is continuous in Y for $u \in Y$, then $U(t,s)$ satisfies the stronger differentiability property
$$\frac{\partial U}{\partial t}(t,s)u = L(t)U(t,s)u, \quad 0 \le s < t \le T.$$
Consequently equation (3) *has a unique Y-valued solution given by $U(t,s)u_0 = u(t)$.*

2.2 Scales of Banach Spaces

In this section we will introduce the notion of a one-parameter family of Banach spaces and state some consequences for the corresponding nonautonomous Cauchy problems, which will be useful later.

Definition 2.3. A scale of Banach spaces of type 1 is a one-parameter family $(\mathbb{B}_\alpha, \|\cdot\|_\alpha)_{\alpha_* \leq \alpha \leq \alpha^*}$ with $\alpha_* < \alpha^*$ satisfying

$$\alpha' < \alpha \Longrightarrow \|\cdot\|_\alpha \leq \|\cdot\|_{\alpha'}, \quad \mathbb{B}_{\alpha'} \subset \mathbb{B}_\alpha.$$

Analogous, a scale of Banach spaces of type 2 is a one-parameter family $(\mathbb{B}'_\alpha, \|\cdot\|_\alpha)_{\alpha_* \leq \alpha \leq \alpha^*}$ with

$$\alpha' < \alpha \Longrightarrow \|\cdot\|_{\alpha'} \leq \|\cdot\|_\alpha, \quad \mathbb{B}'_\alpha \subset \mathbb{B}'_{\alpha'}.$$

$\mathbb{B}_\alpha$ will always denote a scale of Banach spaces of type 1 and $\mathbb{B}'_\alpha$ a scale of type 2.

The family of weighted L^p spaces is a natural example for scales of Banach spaces. Let (Ω, μ) be a measurable space and $\omega : \Omega \longrightarrow \mathbb{R}_+$ be a measurable function. Define the weighted L^p spaces by

$$\mathbb{B}_\alpha = \left\{ f : \Omega \longrightarrow \mathbb{K} : \quad \|f\|_\alpha^p = \int_\Omega |f(x)|^p e^{-\alpha\omega(x)} \mathrm{d}\mu(x) < \infty \right\}$$

for $1 \leq p < \infty$ and for $p = \infty$ as the weighted Banach space with the norm

$$\|f\|_\alpha = \operatorname*{ess\,sup}_{x \in \Omega} |f(x)| e^{-\alpha\omega(x)}.$$

Clearly $(\mathbb{B}_\alpha, \|\cdot\|_\alpha)$ is a scale of Banach spaces of type 1 and $\mathbb{B}'_\alpha = \mathbb{B}_{-\alpha}$ a scale of Banach spaces of type 2.

Remark 2.2.

1. We do not impose conditions whether the embeddings from the smaller into the bigger Banach spaces are dense. In applications we will consider the scale of L^1- respectively L^∞-type spaces, so this condition would not hold for $\mathbb{B}_\alpha$. In [2] the author uses the density of embeddings to prove some sufficient conditions for the well-posedness of equation (3).

2. In general, the spaces $\mathbb{B}_\alpha$, $\bigcup_{\alpha' < \alpha} \mathbb{B}_{\alpha'}$ and $\bigcap_{\alpha'' > \alpha} \mathbb{B}_{\alpha''}$ are different for a scale of type 1. The same is valid for a scale of type 2.

Using this approach, one has the possibility to overcome the difficulty of the time dependence of the domain $D(L(t))$. More precisely one would like to consider the operators $L(t)$ as bounded operators acting from smaller into a bigger Banach space, cf. [2, 5, 11]. Using this, one could consider the operators $(L(t), \mathrm{Dom})_{t \in [0,T]}$ on $\mathbb{B}_\alpha$ with the domain

$$\mathrm{Dom} = \bigcup_{\alpha' < \alpha} \mathbb{B}_{\alpha'}$$

for a scale of type 1. In the case of a scale of type 2 one has $\mathrm{Dom} = \bigcup_{\alpha<\alpha'} \mathbb{B}'_{\alpha'}$. Except for Theorem 2.1 and 2.3 we do not need any conditions of closedness of the operators for this approach. Unfortunately, the solutions will only exist on a bounded time interval $[0, T_*)$. As a consequence of the proof we will see that the solutions evolve in this scale of Banach spaces.

Now assume $Y \hookrightarrow X$ are Banach spaces and $I \ni t \longrightarrow L(t) \in L(Y, X)$ is strongly continuous. Then $\sup_{t\in I} \|L(t)u\|_X < \infty$ holds for all $u \in Y$ and by Banach-Steinhaus theorem $L(t)$ is uniformly bounded in $t \in I$, i.e. $M := \sup_{t\in I} \|L(t)\|_{L(Y,X)} < \infty$. Moreover, for each function $u \in C([0, T]; Y)$ the mapping $I \ni t \longmapsto L(t)u(t) \in X$ is continuous, which follows for $t_0, t \in I$ from

$$\begin{aligned} &\|L(t)u(t) - L(t_0)u(t_0)\|_X \\ \leq\ &\|L(t)u(t) - L(t)u(t_0)\|_X + \|L(t)u(t_0) - L(t_0)u(t_0)\|_X \\ \leq\ &M\|u(t) - u(t_0)\|_Y + \|L(t)u(t_0) - L(t_0)u(t_0)\|_X. \end{aligned}$$

For our calculations, we will need the following product formula for evolution families, which proof shall be omitted.

Lemma 2.3. *Let $Y \hookrightarrow X$ be Banach spaces, $U : \Delta \longrightarrow L(X)$ strongly continuous in the second variable for fixed $t \in I$ and let $s \longmapsto U(t, s)u \in X$ be continuously differentiable for fixed $t \in I$ and $u \in Y$. Then for each $u \in C^1(I, X)$ such that $u(t) \in Y$ with $t \in I$ the equation*

$$\frac{\partial}{\partial s}\left(U(t, s)u(s)\right) = \frac{\partial U}{\partial s}(t, s)u(s) + U(t, s)\frac{\partial u}{\partial s}(s), \quad (t, s) \in \Delta \tag{4}$$

holds on X.

Remark 2.3.

1. Of course, we can apply this lemma for strongly continuously differentiable evolution families as in Theorem 2.1 and 2.3.

2. In many applications the so-called exponential growth condition

   $$\|U(t, s)\|_{L(X)} \leq Ce^{\omega(t-s)}$$

   is satisfied. Nevertheless there are evolution families that do not have exponential growth. For example denote by X the space of all continuous bounded functions $f : \mathbb{R} \longrightarrow \mathbb{R}$ and let $0 < p : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ be bounded. The expression

   $$U(t, s)f(x) = \frac{p(t)}{p(s)} f(x), \quad x \in \mathbb{R}$$

   defines an operator $U(t, s) \in L(X)$ with $\|U(t, s)\|_{L(X)} = \dfrac{p(t)}{p(s)}$. If p is not bounded away from 0, then clearly $U(t, s)$ cannot be exponentially bounded. Note that a strongly continuous semigroup $(T(t))_{t\geq 0}$ always obeys a bound $\|T(t)\|_{L(X)} \leq Ce^{\omega t}$.

2.3 The Space of Solutions

At first, we will give a formal definition of a solution to equation (3) in a scale of Banach spaces. The idea is to consider solutions in some Banach space $\mathbb{B}_{\alpha^*}$ with the property that for each t there exist α_t such that $u(t) \in \mathbb{B}_{\alpha_t}$ holds. Additionally we would like to have the differentiability property for each α in the space $\mathbb{B}_\alpha$. In other words a solution is a consistent family of solutions in the spaces $\mathbb{B}_\alpha$.

Definition 2.4. Given a scale of Banach spaces of type 1 and $L(t) \in L(\mathbb{B}_{\alpha'}, \mathbb{B}_\alpha)$ for $\alpha' < \alpha$ and $t \in [0,T]$. A solution in the scale $\mathbb{B}_\alpha$ to the Cauchy problem

$$\frac{\partial u}{\partial t}(t) = L(t)u(t), \quad u(0) = u_0 \in \mathbb{B}_{\alpha_*}, \quad t \in [0,T] \tag{5}$$

is given by a continuous, monotonically increasing function $(\alpha_*, \alpha^*] \ni \alpha \longmapsto T(\alpha) > 0$ with $T(\alpha) \leq T$, which we will call time data, and an element

$$u \in C^1([0, T(\alpha^*)); \mathbb{B}_{\alpha^*})$$

satisfying $u(0) = u_0$ and for all $\alpha \in (\alpha_*, \alpha^*]$ we have

$$u_\alpha := u|_{[0,T(\alpha))} \in C^1([0, T(\alpha)); \mathbb{B}_\alpha) \tag{6}$$

and

$$\frac{\partial u_\alpha}{\partial t}(t) = L(t)u_\alpha(t)$$

in $\mathbb{B}_\alpha$.

Given a scale of Banach spaces of type 2 and $L(t) \in L(\mathbb{B}'_\alpha, \mathbb{B}'_{\alpha'})$ for $\alpha' < \alpha$. A solution in the scale $\mathbb{B}'_\alpha$ to the Cauchy problem

$$\frac{\partial u}{\partial t}(t) = L(t)u(t), \quad u(0) = u_0 \in \mathbb{B}'_{\alpha^*}, \quad t \in [0,T]$$

is given by a continuous, monotonically decreasing function $[\alpha_*, \alpha^*) \ni \alpha \longmapsto T(\alpha) > 0$ with $T(\alpha) \leq T$ and an element

$$u \in C^1([0, T(\alpha_*)); \mathbb{B}'_{\alpha_*})$$

satisfying $u(0) = u_0$ and for all $\alpha \in [\alpha_*, \alpha^*)$ we have

$$u_\alpha := u|_{[0,T(\alpha))} \in C^1([0, T(\alpha)); \mathbb{B}_\alpha)$$

and

$$\frac{\partial u_\alpha}{\partial t}(t) = L(t)u_\alpha(t)$$

in $\mathbb{B}'_\alpha$.

Remark 2.4.

1. The time data $T(\alpha)$ may depend on the initial condition. Nevertheless in our approach this will not be the case.

2. If we start with some given $T(\alpha) > 0$ and unique elements u_α as in (6) satisfying the corresponding equations one can show that $u := u_{\alpha^*}$ is a solution in the scale $\mathbb{B}_\alpha$.

3. The continuity and monotonicity of $T(\alpha)$ implies that for $t \in [0, T(\alpha))$ there exists some $\alpha' < \alpha$ such that $0 \leq t \leq T(\alpha') \leq T(\alpha)$ holds. Thus one has $u_\alpha(t) \in \mathbb{B}_{\alpha'}$ and hence $L(t)u_\alpha(t)$ is well-defined as an element in $\mathbb{B}_\alpha$.

It is possible to rewrite the problem (5) in the integral form

$$u(t) = u_0 + \int_0^t L(\tau)u(\tau)\mathrm{d}\tau,$$

which proof shall be omitted.

Lemma 2.4. *Assume that* $[0, T] \ni t \longmapsto L(t) \in L(\mathbb{B}_{\alpha'}, \mathbb{B}_\alpha)$ *is strongly continuous for any* α, α' *with* $\alpha_* \leq \alpha' < \alpha \leq \alpha^*$*, then the following statements are equivalent:*

1. *u is the solution to* (5) *in the scale* $\mathbb{B}_\alpha$ *with the time data* $T(\alpha)$

2. $u \in C([0, T(\alpha)); \mathbb{B}_\alpha)$ *for all* $\alpha \in (\alpha_*, \alpha^*]$ *and solves*

$$u(t) = u_0 + \int_0^t L(\tau)u(\tau)\mathrm{d}\tau \in \mathbb{B}_\alpha, \quad u_0 \in \mathbb{B}_{\alpha_*} \tag{7}$$

for $t \in [0, T(\alpha))$*, where* $T(\alpha) \leq T$ *is continuous and monotonically increasing.*

With the help of Lemma 2.8 it is easy to show the existence of a solution to equation (5) on a bounded time interval. Assume $\|L(t)\|_{\alpha'\alpha} \leq \dfrac{M}{\alpha - \alpha'}$ for $\alpha' < \alpha$ and that $[0, T] \ni t \longmapsto L(t) \in L(\mathbb{B}_{\alpha'}, \mathbb{B}_\alpha)$ is strongly continuous, where $\| \cdot \|_{\alpha'\alpha}$ denotes the operator norm on $L(\mathbb{B}_{\alpha'}, \mathbb{B}_\alpha)$. We will show this only for a scale of type 1, since the other case can be shown analogous. Let $u_0 \in \mathbb{B}_{\alpha_*}$ and define the sequence

$$u_0(t) = u_0, \quad u_{n+1}(t) = u_0 + \int_0^t L(\tau)u_n(\tau)\mathrm{d}\tau, \quad n \in \mathbb{N}_0, \tag{8}$$

which satisfies

$$u_n(t) = u_0 + \sum_{k=1}^{n} \int_0^t \int_0^{t_1} \dots \int_0^{t_{k-1}} L(t_1)\dots L(t_k)u_0 \mathrm{d}t_k \dots \mathrm{d}t_1 \in C([0, T(\alpha)); \mathbb{B}_\alpha).$$

For $n \in \mathbb{N}$ and $\alpha_* < \alpha < \alpha^*$ define

$$\varepsilon = \frac{\alpha - \alpha_*}{n} \text{ and } \alpha_j = \alpha_* + j\varepsilon \text{ for } j = 0, \dots, n, \tag{9}$$

so we have $\alpha_0 = \alpha_*$, $\alpha_n = \alpha$ and $\alpha_{j+1} - \alpha_j = \varepsilon$ and hence

$$\begin{aligned} \|u_n(t) - u_{n-1}(t)\|_\alpha &\leq \int_0^t \int_0^{t_1} \dots \int_0^{t_{n-1}} \left(\frac{M}{\varepsilon}\right)^n \|u_0\|_{\alpha_*} \mathrm{d}t_n \dots \mathrm{d}t_1 \\ &= \frac{1}{n!} \left(\frac{tnM}{\alpha - \alpha_*}\right)^n \|u_0\|_{\alpha_*}. \end{aligned} \tag{10}$$

Using Stirlings formula we see that the right hand side is summable in $n \in \mathbb{N}$ for $|t| < T(\alpha)$ with

$$T(\alpha) = \frac{\alpha - \alpha_*}{eM}. \tag{11}$$

Hence, $(u_n(t))_{n\in\mathbb{N}} \subset \mathbb{B}_\alpha$ is a fundamental sequence and therefore has a limit $u(t) = \lim\limits_{n\to\infty} u_n(t) \in \mathbb{B}_\alpha$ for $t \in [0, T(\alpha))$. Moreover, the convergence is uniform on each interval $[0, s] \subset [0, T(\alpha))$. To show this, consider for $n < m$

$$\|u_m(t) - u_n(t)\|_\alpha \leq \sum_{k=n}^{m-1} \|u_{k+1}(t) - u_k\|_\alpha \leq \text{Const.} \sum_{k=n}^{\infty} \left(\frac{t}{T(\alpha)}\right)^k$$

and obtain by passing to the limit $m \to \infty$

$$\|u_n(t) - u(t)\|_\alpha \leq \text{Const.} \sum_{k=n}^{\infty} \left(\frac{t}{T(\alpha)}\right)^k.$$

Therefore $u \in C([0, T(\alpha)); \mathbb{B}_\alpha)$ and by

$$\begin{aligned} \|L(t)u_n(t) - L(t)u_{n-1}(t)\|_\alpha &\leq \left(\frac{M}{\varepsilon}\right)^{n+1} \|u_0\|_{\alpha_*} \frac{t^n}{n!} \\ &= \frac{nM}{\alpha - \alpha_*} \|u_0\|_{\alpha_*} \frac{1}{n!} \left(\frac{tnM}{\alpha - \alpha_*}\right)^n \end{aligned}$$

the convergence $L(t)u_n(t) \to L(t)u(t)$ holds uniformly on compact intervals $t \in [0, s] \subset [0, T(\alpha))$. Consequently taking the limit in (8) we obtain equation (7).

Remark 2.5.

1. In the same way one can show the existence for arbitrary initial times t_0. In this case we would have the condition $|t - t_0| < T(\alpha)$ for convergence.

2. The difficulty is to show that the solution above is unique. Our assumptions on $L(t)$ do not allow to apply the Gronwall Lemma. To overcome this difficulty we will solve the corresponding integral equation (7) in some Banach space S^β, which reflects the properties of a solution in a scale $\mathbb{B}_\alpha$.

The general result for a quasilinear Cauchy problem in a scale of type 2 was published by Safonov in 1995 in [26]. Here we will only present a proof for the linear equation in a scale of type 1. The last result suggests that $T(\alpha) = \dfrac{\alpha - \alpha_*}{\lambda}$ for $\lambda > 0$ is a natural candidate for the time data. This motivates the following definition.

Definition 2.5. For $\lambda > 0$ and $\beta \geq 0$ let

$$S_1^\beta(\alpha_*, \alpha^*, \lambda) \equiv S_1^\beta = \left\{ u \in \bigcap_{\alpha_* \leq \alpha \leq \alpha^*} C\left(\left[0, \frac{\alpha - \alpha_*}{\lambda}\right); \mathbb{B}_\alpha \right) \,\middle|\, \|u\|_1^{(\beta)} < \infty \right\}$$

for the type 1 scale and

$$S_2^\beta(\alpha_*, \alpha^*, \lambda) \equiv S_2^\beta = \left\{ u \in \bigcap_{\alpha_* \leq \alpha \leq \alpha^*} C\left(\left[0, \frac{\alpha^* - \alpha}{\lambda}\right); \mathbb{B}'_\alpha \right) \,\middle|\, \|u\|_2^{(\beta)} < \infty \right\}$$

for the type 2 scale. The norms are given by

$$\begin{aligned} \|u\|_1^{(\beta)} &= \sup_{\alpha \in [\alpha_*, \alpha^*],\ t \in [0, T_1(\alpha))} (\alpha - \alpha_* - \lambda t)^\beta \|u(t)\|_\alpha \\ \|u\|_2^{(\beta)} &= \sup_{\alpha \in [\alpha_*, \alpha^*],\ t \in [0, T_2(\alpha))} (\alpha^* - \alpha - \lambda t)^\beta \|u(t)\|_\alpha \end{aligned}$$

with $T_1(\alpha) = \dfrac{\alpha - \alpha_*}{\lambda}$ and $T_2(\alpha) = \dfrac{\alpha^* - \alpha}{\lambda}$.

Here we use the notation $C([0,0); \mathbb{B}_{\alpha_*}) = \mathbb{B}_{\alpha_*}$ and $C([0,0); \mathbb{B}_{\alpha^*}) = \mathbb{B}_{\alpha^*}$.

Clearly this spaces are complete and therefore Banach spaces.

2.4 Existence of local solutions and properties

In the main part of this section we will discuss two possibilities to show existence of solutions to (5). The first existence result is a simplified version of the general result from [26].

Theorem 2.5. *Consider a scale* $(\mathbb{B}_\alpha, \|\cdot\|_\alpha)_{\alpha_* \leq \alpha \leq \alpha^*}$ *of type 1 and assume that there exist* $\lambda_a > 0$ *and* $M \geq 0$ *such that*

1. $\left[0, \dfrac{\alpha^* - \alpha_*}{\lambda_a}\right) \ni t \longmapsto L(t) \in L(\mathbb{B}_{\alpha'}, \mathbb{B}_\alpha)$ *is strongly continuous for any* $\alpha' < \alpha$
2. $\|L(t)\|_{\alpha'\alpha} \leq \dfrac{M}{\alpha - \alpha'}$ *for any* $\alpha' < \alpha$ *and* $t \in \left[0, \dfrac{\alpha^* - \alpha_*}{\lambda_a}\right)$.

Then there exists $\lambda_0 > \lambda_a > 0$ *and* $T_\lambda : (\alpha_*, \alpha^*] \longrightarrow \mathbb{R}_+$ *continuous and monotonically increasing given by*

$$T_\lambda(\alpha) = \frac{\alpha - \alpha_*}{\lambda}, \quad \text{with } \lambda > \lambda_0$$

such that for each initial condition $u_0 \in \mathbb{B}_{\alpha_*}$ *there exists a unique solution* $u \in S_1^\beta(\lambda)$ *to the Cauchy problem*

$$\frac{\partial u}{\partial t}(t) = L(t)u(t), \quad u(0) = u_0$$

in the scale $\mathbb{B}_\alpha$.

Assuming we have proved this theorem, we can also state the following version.

Theorem 2.6. *Consider the type 2 scale* $(\mathbb{B}'_\alpha, \|\cdot\|_\alpha)_{\alpha_* \le \alpha \le \alpha^*}$ *and assume that there exist* $\lambda_a > 0$ *and* $M \ge 0$ *such that*

1. $\left[0, \frac{\alpha^* - \alpha_*}{\lambda_a}\right) \ni t \longmapsto L(t) \in L(\mathbb{B}'_\alpha, \mathbb{B}'_{\alpha'})$ *is strongly continuous for any* $\alpha' < \alpha$
2. $\|L(t)\|_{\alpha\alpha'} \le \frac{M}{\alpha - \alpha'}$ *for any* $\alpha' < \alpha$ *and* $t \in \left[0, \frac{\alpha^* - \alpha_*}{\lambda_a}\right)$.

Then there exists $\lambda_0 > \lambda_a > 0$ *and* $T_\lambda : [\alpha_*, \alpha^*) \longrightarrow \mathbb{R}_+$ *continuous and monotonically decreasing given by*

$$T_\lambda(\alpha) = \frac{\alpha^* - \alpha}{\lambda}, \quad \textit{with } \lambda > \lambda_0$$

such that for each initial condition $u_0 \in \mathbb{B}'_{\alpha^*}$ *there exists a unique solution* $u \in S_2^\beta(\lambda)$ *to the Cauchy problem*

$$\frac{\partial u}{\partial t}(t) = L(t)u(t), \quad u(0) = u_0$$

in the scale $\mathbb{B}_\alpha$.

Proof. Define the spaces $\widetilde{\mathbb{B}}_\alpha = \mathbb{B}'_{\alpha_* + \alpha^* - \alpha}$ with the norm $\|\cdot\|'_\alpha = \|\cdot\|_{\alpha_* + \alpha^* - \alpha}$ for $\alpha_* \le \alpha \le \alpha^*$ and apply the first result. □

Now we will prove the first stated version, namely Theorem 2.11.

Proof. By Lemma 2.8 it is enough to solve the equation

$$u(t) = u_0 + \int_0^t L(\tau)u(\tau)\mathrm{d}\tau =: u_0 + (Tu)(t)$$

in the space S^γ. So let $\lambda \ge \lambda_a$ and $\beta > 0$. To abuse notation, we will write in this proof $\|\cdot\|^{(\beta)}$ for the norm $\|\cdot\|_1^{(\beta)}$.

1. For $u \in S^\beta$ we have: $\|L(\cdot)u(\cdot)\|^{(\beta+1)} \le M2^{\beta+1}\|u\|^{(\beta)}$.

   Indeed, let $0 \le t < \frac{\alpha - \alpha_*}{\lambda}$ and take $\alpha' < \alpha$ so close to α that we have $0 \le t < \frac{\alpha' - \alpha_*}{\lambda} < \frac{\alpha - \alpha_*}{\lambda}$. Thus $u(t) \in \mathbb{B}_{\alpha'}$ implies $L(t)u(t) \in \mathbb{B}_\alpha$ and since α and t were arbitrary we obtain

   $$L(t)u(t) \in \mathbb{B}_\alpha, \quad 0 \le t < \frac{\alpha - \alpha_*}{\lambda}.$$

   Now let $\alpha \in (\alpha_*, \alpha^*]$ and $t \in [0, T_\lambda(\alpha))$ be arbitrary and define

   $$\rho = \alpha - \alpha_* - \lambda t, \quad \alpha' = \alpha - \frac{\rho}{2}.$$

   For such ρ and α the following holds

(a) $\alpha_* \le \alpha' < \alpha \le \alpha^*$

(b) $\alpha - \alpha' = \dfrac{\rho}{2} = \alpha - \alpha_* - \lambda t - \dfrac{\rho}{2} = \alpha' - \alpha_* - \lambda t$

(c) $\alpha - \alpha_* - \lambda t = \rho = 2(\alpha' - \alpha_* - \lambda t)$

and hence we obtain

$$\begin{aligned}(\alpha - \alpha_* - \lambda t)^{\beta+1}\|L(t)u(t)\|_\alpha &\le M\frac{(\alpha-\alpha_*-\lambda t)^{\beta+1}}{\alpha-\alpha'}\|u(t)\|_{\alpha'} \\ &= M2^{\beta+1}(\alpha'-\alpha_*-\lambda t)^\beta\|u(t)\|_{\alpha'} \\ &\le M2^{\beta+1}\|u\|^{(\beta)}\end{aligned}$$

which implies $\|L(\cdot)u(\cdot)\|^{(\beta+1)} \le M2^{\beta+1}\|u\|^{(\beta)}$.

2. For $u \in S^{\beta+1}$ we have: $\|Tu\|^{(\beta)} \le \dfrac{M2^{\beta+1}}{\beta\lambda}\|u\|^{(\beta+1)}$.

   Indeed, let $\alpha_* \le \alpha \le \alpha^*$ and $t \in [0, T_\lambda(\alpha))$, then we have

$$\begin{aligned}\left\|\int_0^t u(\tau)\mathrm{d}\tau\right\|_\alpha &\le \int_0^t \|u(\tau)\|_\alpha \mathrm{d}\tau \le \int_0^t (\alpha-\alpha_*-\lambda\tau)^{-\beta-1}\mathrm{d}\tau\|u\|^{(\beta+1)} \\ &\le \frac{\|u\|^{(\beta+1)}}{\beta\lambda}(\alpha-\alpha_*-\lambda t)^{-\beta}\end{aligned}$$

   and so $\left\|\int_0^{\bullet} u(\tau)\mathrm{d}\tau\right\|^{(\beta)} \le \dfrac{\|u\|^{(\beta+1)}}{\beta\lambda}$. Now the statement follows from

$$\|Tu\|^{(\beta)} = \left\|\int_0^{\bullet} L(\tau)u(\tau)\mathrm{d}\tau\right\|^{(\beta)} \le \frac{1}{\beta\lambda}\|L(\cdot)u(\cdot)\|^{(\beta+1)} \le \frac{M2^{\beta+1}}{\beta\lambda}\|u\|^{(\beta)}.$$

3. We saw that for all $\lambda > \max\{\lambda_a, \dfrac{M2^{\beta+1}}{\beta}\} =: \lambda_0 \ge \lambda_a$

$$\|Tu\|^{(\beta)} \le \frac{\lambda_0}{\lambda}\|u\|^{(\beta)} < \|u\|^{(\beta)}$$

   holds. Let $u_0 \in \mathbb{B}_{\alpha_*}$ be arbitrary. Using

$$\begin{aligned}\|u_0\|^{(\beta)} &= \sup_{\alpha\in[\alpha_*,\alpha^*],\ t\in[0,T_\lambda(\alpha))} (\alpha-\alpha_*-\lambda t)^\beta\|u_0\|_\alpha \\ &\le \left(\sup_{\alpha\in[\alpha_*,\alpha^*],\ t\in[0,T_\lambda(\alpha))} (\alpha-\alpha_*-\lambda t)^\beta\right)\|u_0\|_{\alpha_*}\end{aligned}$$

   one sees that $u_0 \in S^\beta$ and hence the sequence $(u^{(k)})_{k\in\mathbb{N}}$ given by $u^{(0)} = u_0$ and $u^{(k+1)} = u_0 + Tu^{(k)}$ satisfies $u^{(k)} \in S^\beta$, cf. Definition 5. Due to

$$\|u^{(k+1)} - u^{(k)}\|^{(\beta)} \le \left(\frac{\lambda_0}{\lambda}\right)^k \|u^{(1)} - u^{(0)}\|^{(\beta)}$$

this sequence has a limit $u \in S^\beta$. By definition this limit satisfies

$$u(t) = u_0 + \int_0^t L(\tau)u(\tau)\mathrm{d}\tau, \quad \forall t \in [0, T_\lambda(\alpha))$$

with the time data $T_\lambda(\alpha) = \dfrac{\alpha - \alpha_*}{\lambda}$.

4. For uniqueness let $v \in S^\beta$ solves the Cauchy problem with the zero initial data or equivalently

$$v(t) = \int_0^t L(\tau)v(\tau)\mathrm{d}\tau = (Tv)(t).$$

So v is a fix-point of T and because of $\|T\|_{L(S^\beta)} \leq \dfrac{\lambda_0}{\lambda} < 1$ we have that $v = 0$. □

Remark 2.6.

1. Minimizing the expression $\dfrac{2^\beta}{\beta}$ we obtain for $\beta = \dfrac{1}{\log(2)}$ and λ_a small

$$\lambda_0 = 2eM\log(2) = eM\log(4).$$

So up to the factor log(4) this is the same time data as in the first existence result, cf. (11).

2. Note $u(t) \in \bigcap\limits_{\alpha > \alpha_t} \mathbb{B}_\alpha$, where α_t is given by

$$0 \leq t < \frac{\alpha_t - \alpha_*}{\lambda} \Longleftrightarrow 0 \leq \alpha_* + \lambda t < \alpha_t.$$

Thus we have $u(t) \in \bigcap\limits_{\alpha > \alpha_* + \lambda t} \mathbb{B}_\alpha$.

3. Now we have solved the Cauchy problem for each $\beta > 0$ and $\lambda > \lambda_0$, so there are solutions $u = u_{\beta,\lambda}$. For each $\lambda > \lambda_0$ and $\beta' < \beta$ the inequality

$$(\alpha - \alpha_* - \lambda t)^\beta = (\alpha - \alpha_* - \lambda t)^{\beta'}(\alpha - \alpha_* - \lambda t)^{\beta - \beta'} \leq (\alpha^* - \alpha_*)^{\beta - \beta'}(\alpha - \alpha_* - \lambda t)^{\beta'}$$

implies $\|\cdot\|_1^\beta \leq (\alpha^* - \alpha_*)^{\beta - \beta'}\|\cdot\|_1^{\beta'}$. The same holds for a scale of type 2. Consequently we obtain $S^{\beta'} \subset S^\beta$ for each $\beta' < \beta$. Since λ_0 depends on β we use Remark 2.12.1 and chose $\beta = \dfrac{1}{\log(2)}$ to obtain solutions on the biggest possible time interval. But in the same way

$$(\alpha - \alpha_* - \lambda t)^\beta \leq (\alpha - \alpha_* - \lambda' t)^\beta$$

for $\lambda' < \lambda$ implies that the solutions satisfy

$$u_{\beta,\lambda}(t) = u_{\beta,\lambda'}(t) \quad \text{for } t \in [0, T_\lambda(\alpha)) \subset [0, T_{\lambda'}(\alpha)).$$

So taking $\beta = \dfrac{1}{\log(2)}$ and $T(\alpha) = \dfrac{\alpha - \alpha_*}{\lambda_0}$ we obtain the existence of a unique solution $u : \left[0, \dfrac{\alpha^* - \alpha_*}{\lambda_0}\right) \longrightarrow \mathbb{B}_{\alpha^*}$. By construction each restriction to $[0, T_\lambda(\alpha)]$ corresponds to some element

$$u|_{[0,T_\lambda(\alpha)]} \in C^1\left([0, T_\lambda(\alpha)]; \mathbb{B}_\alpha\right)$$

for $\lambda > \lambda_0$ and $\sup\limits_{\alpha\in[\alpha_*,\alpha^*], t\in[0,T_\lambda(\alpha))} (\alpha - \alpha_* - \lambda t)^\beta \|u\|_\alpha < \infty$ solving the Cauchy problem in $\mathbb{B}_\alpha$.

Now let $\lambda > \lambda_0$, $u_0, v_0 \in \mathbb{B}_{\alpha_*}$ be two initial conditions and u respectively v the corresponding solutions. Then we have

$$\|u - v\|^{(\beta)} \leq \|u_0 - v_0\|^{(\beta)} + \|T(u - v)\|^{(\beta)} \leq \|u_0 - v_0\|^{(\beta)} + \frac{\lambda_0}{\lambda}\|u - v\|^{(\beta)}$$

and hence

$$\|u - v\|^{(\beta)} \leq \frac{\lambda}{\lambda - \lambda_0}\|u_0 - v_0\|^{(\beta)}.$$

Taking into account that

$$\begin{aligned} \|u_0 - v_0\|^{(\beta)} &= \sup_{\alpha\in[\alpha_*,\alpha^*],\ t\in[0,T_\lambda(\alpha))} (\alpha - \alpha_* - \lambda t)^\beta \|u_0 - v_0\|_\alpha \\ &\leq (\alpha^* - \alpha_*)^\beta \|u_0 - v_0\|_{\alpha_*} \end{aligned}$$

we can rewrite

$$\|u - v\|^{(\beta)} \leq \frac{\lambda}{\lambda - \lambda_0}(\alpha^* - \alpha_*)^\beta \|u_0 - v_0\|_{\alpha_*}$$

or using $\alpha \in (\alpha_*, \alpha^*]$ and $t \in [0, T_\lambda(\alpha))$

$$\|u(t) - v(t)\|_\alpha \leq \frac{\lambda}{\lambda - \lambda_0}\left(\frac{\alpha^* - \alpha_*}{\alpha - \alpha_* - \lambda t}\right)^\beta \|u_0 - v_0\|_{\alpha_*}.$$

This shows, that the solutions depend continuously on the initial data u_0, v_0. It is possible to show a stronger result, but this part shall be omitted. Now we would like to handle the situation, where $L(t)$ does not satisfy an estimate $\|L(t)\|_{\alpha'\alpha} \leq \dfrac{M}{\alpha - \alpha'}$. In applications effects like pair interaction lead to operators, which do not satisfy above estimate. Nevertheless the following approach may be still applicable. Assume $L(t)$ can be decomposed into $L(t) = A(t) + B(t)$, where $B(t)$ still satisfies this assumption. If we can solve the Cauchy problem for $A(t)$ with an evolution family, one can try to solve the Cauchy problem for $L(t)$ using similar arguments like the ones before. This approach is realized in the next theorem.

Theorem 2.7. *Let $(\mathbb{B}_\alpha, \|\cdot\|_\alpha)_{\alpha_*\leq\alpha\leq\alpha^*}$ be a scale of type 1 and $\lambda_a > 0$ such that $A(t)$ satisfies the following assumptions*

1. *For all α', α with $\alpha_* \leq \alpha' < \alpha \leq \alpha^*$ the mapping $[0, \dfrac{\alpha^* - \alpha_*}{\lambda_a}] \ni t \longmapsto A(t) \in L(\mathbb{B}_{\alpha'}, \mathbb{B}_\alpha)$ is strongly continuous,*

2. *For all* $\alpha \in [\alpha_*, \alpha^*]$ *there exists an evolution family* $U : \Delta \longrightarrow L(\mathbb{B}_\alpha)$ *such that* $\|U(t,s)\|_{L(\mathbb{B}_\alpha)} \leq 1$ *for* $(t,s) \in \Delta = \{(t,s) \in [0, \frac{\alpha^* - \alpha_*}{\lambda_a}]^2 : \ s \leq t\}$,

3. *For all* $\alpha' < \alpha$ *and* $u \in \mathbb{B}_{\alpha'}$

$$\Delta \ni (t,s) \longmapsto U(t,s)u \in \mathbb{B}_\alpha$$

is differentiable with derivatives

$$\frac{\partial U}{\partial t}(t,s)u = A(t)U(t,s)u, \quad 0 \leq s \leq t \leq \frac{\alpha^* - \alpha_*}{\lambda_a}$$

and

$$\frac{\partial U}{\partial s}(t,s)u = -U(t,s)A(s)u, \quad 0 \leq s \leq t \leq \frac{\alpha^* - \alpha_*}{\lambda_a}.$$

In the case of $s = t$ *the derivative* $\frac{\partial U}{\partial t}(t,s)u$ *is to be understood as a right-sided derivative.*

Further assume that $[0, \frac{\alpha^* - \alpha_*}{\lambda_a}] \ni t \longmapsto B(t) \in L(\mathbb{B}_{\alpha'}, \mathbb{B}_\alpha)$ *is strongly continuous for all* $\alpha' < \alpha$ *satisfying* $\|B(t)\|_{\alpha'\alpha} \leq \frac{M}{\alpha - \alpha'}$. *Then there exists* $\lambda_0 > \lambda_a > 0$ *and* $T_\lambda : (\alpha_*, \alpha^*] \longrightarrow \mathbb{R}_+$ *continuous and monotonically increasing given by*

$$T_\lambda(\alpha) = \frac{\alpha - \alpha_*}{\lambda} \quad \text{with } \lambda > \lambda_0$$

such that for each initial condition $u_0 \in \mathbb{B}_{\alpha_*}$ *there exists a unique solution* u *in* $S^\beta(\lambda)$ *to the Cauchy problem*

$$\frac{\partial u}{\partial t}(t) = (A(t) + B(t))u(t), \quad u(0) = u_0 \tag{12}$$

in the scale $\mathbb{B}_\alpha$.

Analogous to the previous result the first step is to reformulate the Cauchy problem in the integral form. This will be the content of the next lemma, for which (4) is needed.

Lemma 2.8. *Let* $A(t), B(t), U(t,s)$ *be like in Theorem 2.13. Then the following statements are equivalent:*

1. u *is a solution to* (12) *in the scale* $(\mathbb{B}_\alpha, \|\cdot\|_\alpha)_{\alpha_* \leq \alpha \leq \alpha^*}$ *with a time data* $T(\alpha) > 0$

2. $u \in \bigcap_{\alpha_* \leq \alpha \leq \alpha^*} C([0, T(\alpha)); \mathbb{B}_\alpha)$ *solves the equation*

$$u(t) = U(t,0)u_0 + \int_0^t U(t,\tau)B(\tau)u(\tau)\mathrm{d}\tau \tag{13}$$

in $\mathbb{B}_\alpha$ *for* $t \in [0, T(\alpha))$ *and* $\alpha \in (\alpha_*, \alpha^*]$.

Using Lemma 2.14 we are now in a position to prove Theorem 2.13 in a scale of Banach spaces of type 1.

Proof. For $\lambda > \lambda_a$ we will solve the equation (13)

$$u(t) = U(t,0)u_0 + \int\limits_0^t U(t,\tau)B(\tau)u(\tau)\mathrm{d}\tau = U(t,0)u_0 + (Tu)(t).$$

Write $\|\cdot\|^{(\beta)}$ and T_λ as before. Using $\|U(t,s)\|_{L(\mathbb{B}_\alpha)} \leq 1$ we obtain for $u \in S^{\beta+1}$, $\alpha \in [\alpha_*, \alpha^*]$ and $t \in [0, T_\lambda(\alpha))$ by the proof of Theorem 2.10

$$\left\| \int\limits_0^t U(t,\tau)u(\tau)\mathrm{d}\tau \right\|_\alpha \leq \int\limits_0^t \|u(\tau)\|_\alpha \mathrm{d}\tau \leq \frac{(\alpha - \alpha_* - \lambda t)^{-\beta}}{\beta\lambda}\|u\|^{(\beta+1)}.$$

As a result we have shown $\left\| \int\limits_0^{\bullet} U(\cdot,\tau)u(\tau)\mathrm{d}\tau \right\|^{(\beta)} \leq \frac{1}{\beta\lambda}\|u\|^{(\beta+1)}$ and therefore

$$\|Tu\|^{(\beta)} = \left\| \int\limits_0^{\bullet} U(\cdot,\tau)B(\tau)u(\tau)\mathrm{d}\tau \right\|^{(\beta)} \leq \frac{1}{\beta\lambda}\|B(\cdot)u(\cdot)\|^{(\beta+1)} \leq \frac{M2^{\beta+1}}{\beta\lambda}\|u\|^{(\beta)}.$$

For the same λ as in the previous proof and $\beta > 0$ we have $\|Tu\|^{(\beta)} \leq \frac{\lambda_0}{\lambda}\|u\|^{(\beta)}$. Now define a sequence by $u^{(0)}(t) = U(t,0)u_0$ and $u^{(k+1)}(t) = U(t,0)u_0 + (Tu^{(k)})(t)$. From

$$\begin{aligned}
\|u^{(0)}\|^{(\beta)} &= \sup_{\alpha\in[\alpha_*,\alpha^*],\quad t\in[0,T_\lambda(\alpha))} (\alpha - \alpha_* - \lambda t)^\beta \|U(t,0)u_0\|_\alpha \\
&\leq \sup_{\alpha\in[\alpha_*,\alpha^*],\quad t\in[0,T_\lambda(\alpha))} (\alpha - \alpha_* - \lambda t)^\beta \|u_0\|_\alpha \\
&\leq (\alpha^* - \alpha_*)^\beta \|u_0\|_{\alpha_*} < \infty
\end{aligned}$$

one easily sees $(u^{(k)})_{k\in\mathbb{N}} \subset S^\beta$. Therefore, $(u^{(k)})_{k\in\mathbb{N}}$ is a fundamental sequence for $\lambda > \lambda_0 = \max\left\{\frac{M2^{\beta+1}}{\beta}, \lambda_a\right\}$ and hence there exists a limit $\lim\limits_{k\to\infty} u^{(k)} = u \in S^\beta$, which solves the equation

$$u = U(\cdot,0)u_0 + Tu$$

by definition, which shows (12). This shows the existence of a solution. For uniqueness let $v \in S^\beta$ be another solution, then $w = u - v$ solves $w = Tw$ and therefore $w = 0$, since T is a contraction. □

Remark 2.7.

1. Under some modifications it is clear that a similar result can be stated for a scale of Banach spaces of type 2.

2. A similar result for the time independent case was stated in [5]. The authors have shown the existence of solutions directly by using (9). To establish uniqueness they have used analyticity at 0 and the formula $\frac{\mathrm{d}^n u}{\mathrm{d}t^n}(0) = L^n u(0)$. Unfortunately such a formula does not hold for the time dependent case and due to the properties of the operators it is not possible to apply the Gronwall Lemma, which is the reason for this approach.

3. The same considerations as in Remark 2.12.3 hold also here. If we weaken the assumption $\|U(t,s)\|_{L(\mathbb{B}_\alpha)} \leq 1$ to

$$\sup_{(t,s)\in\Delta} \|U(t,s)\|_{L(\mathbb{B}_\alpha)} \leq C < \infty$$

with $\Delta = \left\{ (t,s) \in \left[0, \frac{\alpha^* - \alpha_*}{\lambda_a}\right]^2 : \ \ s \leq t \right\}$ and for some constant $C > 0$ independent of α, then a similar result holds. More precisely one has

$$\lambda_0 = \min\left\{ \lambda_a, \frac{MC2^{\beta+1}}{\beta} \right\}$$

and consequently Remark 2.12.3 still holds. Note that the supremum always exists, but in general might be not bounded with respect to α.

Similar to the first version one can show $\|u - v\|^{(\beta)} \leq C\|u_0 - v_0\|^{(\beta)}$ for some constant $C > 0$. Likewise it is possible to show a stronger result concerning continuous dependence of the solutions on parameters. To summarize we have shown the existence of solutions in scales of Banach spaces under the condition that either $\|L(t)\|_{\alpha'\alpha} \leq \frac{M}{\alpha - \alpha'}$ holds or $L(t) = A(t) + B(t)$ satisfies $\|B(t)\|_{\alpha'\alpha} \leq \frac{M}{\alpha - \alpha'}$ and $A(t)$ generates an evolution family. For many applications in interacting particle systems or partial differential equations such results can be used, cf. [26]. For further developments it is useful to construct evolution families under more general assumptions or even using the properties of scales of Banach spaces.

3 Evolutions of interacting particle systems

For motivation we start with an explicit model of interacting particle systems. Consider a habitat with living individuals, e.g. humans, located in $\mathbb{R}^d$. For such individuals we would like to model natural birth and death as elementary events. Now assume that the habitat is contaminated due to some mechanism, i.e. an atomic catastrophy. Hence the individuals will become sick and die according to specific rates. For applications one would like to know how this system will behave in the time evolution. Important questions are concerned with the possibility of whether the individuals would survive this catastrophy or not. To model such a system mathematically we will not distinguish between individuals, meaning that the only important information is the position of the individual. Therefore a population can be described as a subset $\gamma \subset \mathbb{R}^d$. Since we will describe this system in a probabilistic way via Markov evolutions it is enough to give the formal Markov pre-generator. In this case such generator has the form

$$\begin{aligned}(L(t)F)(\gamma) &= \sum_{x\in\gamma}(m(t)+P_t(x))(F(\gamma\backslash x)-F(\gamma))\\ &\quad+\int_{\mathbb{R}^d}\left(\sum_{y\in\gamma}a_t(x-y)\right)(F(\gamma\cup x)-F(\gamma))\mathrm{d}x.\end{aligned}$$

Here and in the further chapters we will just write $\gamma\cup x$ and $\gamma\backslash x$ instead of $\gamma\cup\{x\}$ and $\gamma\backslash\{x\}$ for brevity. The interpretation is that each individual $x\in\gamma$ might die due to a space independent mortality rate $m(t)\geq 0$ and additionally to a space dependent rate $P_t(x)\geq 0$, which describes the habitat. Further each individual located at some point $y\in\mathbb{R}^d$ may produce another individual located at $x\in\mathbb{R}^d$ depending on the time dependent birth rate a_t. In this model the new individual at point $x\in\mathbb{R}^d$ immediately may produce new individuals by themselves. Note that the birth is modeled translation invariant. More generally one can consider a general birth-and-death process given by

$$(L(t)F)(\gamma)=\sum_{x\in\gamma}d_t(x,\gamma\backslash x)(F(\gamma\backslash x)-F(\gamma))+\int_{\mathbb{R}^d}b_t(x,\gamma)(F(\gamma\cup x)-F(\gamma))\mathrm{d}x.$$

A general approach to dynamics on configuration spaces was given in [10] and references therein and [14] contains all necessary technical details for this approach via correlation functions. In the next section we will give a brief outline on general birth and death dynamics on configuration spaces. Afterwards we will use the Sourgailis and continuous Contact model to answer the given questions above. Further sections are devoted to Glauber-type dynamics, Bolkmann-Dieckmann-Law-Pacala model and general birth and death models.

3.1 General Dynamics on Configuration Spaces

The configuration space Γ over $\mathbb{R}^d$ for $d\in\mathbb{N}$ is defined as the set of all locally finite subsets of $\mathbb{R}^d$, i.e.

$$\Gamma=\{\gamma\subset\mathbb{R}^d:\ |\gamma\cap\Lambda|<\infty,\ \ \forall\Lambda\subset\mathbb{R}^d\text{ compact }\}.$$

We will use the notation $\gamma\cap\Lambda=\gamma_\Lambda$ and $|\gamma_\Lambda|$ denotes the cardinality of the set γ_Λ. Denote by $\Gamma_0^{(n)}=\{\gamma\subset\mathbb{R}^d:\ |\gamma|=n\}$ the space of n-point configurations and by

$$\Gamma_0=\bigsqcup_{n=0}^{\infty}\Gamma_0^{(n)}$$

the space of all finite configurations. Via the identification

$$\Gamma\ni\gamma\longmapsto\mathrm{d}\gamma=\sum_{x\in\gamma}\delta_x\in\mathscr{M}(\mathbb{R}^d)$$

one can endow Γ with a topological structure. Here $\mathscr{M}(\mathbb{R}^d)$ stands for the space of all Radon measures on $\mathbb{R}^d$. The topology on Γ is the weakest where all mappings

$$\Gamma \ni \gamma \longmapsto \langle \varphi, \gamma \rangle = \int\limits_{\mathbb{R}^d} \varphi(x) \mathrm{d}\gamma(x) = \sum_{x \in \gamma} \varphi(x) \in \mathbb{R}$$

are continuous for $\varphi \in C_c(\mathbb{R}^d)$. In [25] the author showed that Γ is a polish space and gave a characterization of compact subsets of Γ. It is also possible to define a differentiable structure on Γ and on Γ_0, for further aspects see [1]. Using this differential structure it is possible to prove an integration by parts formula and characterize Gibbs measures, which are the equilibrium states for the Glauber dynamics. The Poisson measure π_z for $z > 0$ is defined as in [1], i.e. as the unique probability measure on $(\Gamma, \mathscr{B}(\Gamma))$ with the Laplace transform

$$\int\limits_{\Gamma} \exp(\langle \varphi, \gamma \rangle) \mathrm{d}\pi_z(\gamma) = \exp\Bigg(z \int\limits_{\mathbb{R}^d} (e^{\varphi(x)} - 1) \mathrm{d}x \Bigg)$$

for $\varphi \in C_c(\mathbb{R}^d)$. It is also possible to define this measure as a projective limit using the Kolmogorov theorem for projective limits. The Lebesgue-Poisson measure λ_z is defined by

$$\lambda_z = \sum_{n=0}^{\infty} \frac{z^n}{n!} m^{(n)} = \delta_{\{\emptyset\}} + \sum_{n=1}^{\infty} \frac{z^n}{n!} m^{(n)},$$

where $m^{(n)}$ is the image measure of the Lebesgue measure $m^{\otimes n}$ on $(\mathbb{R}^d)^n$ under the symmetrization-mapping

$$sym^n : \widetilde{(\mathbb{R}^d)}^n \longrightarrow \Gamma_0^{(n)}, \quad (x_1, \dots, x_n) \longmapsto \{x_1, \dots, x_n\}$$

with $\widetilde{(\mathbb{R}^d)^n} = \{(x_1, \dots, x_n) \in (\mathbb{R}^d)^n : \quad x_j \neq x_k, \text{ with } j \neq k\}$. For $z = 1$ we will write $\lambda = \lambda_1$. We call functions $F : \Gamma \longrightarrow \mathbb{R}$ observables and functions $G : \Gamma_0 \longrightarrow \mathbb{R}$ quasi-observables. The $K-$Transform, given by

$$(KG)(\gamma) = \sum_{\eta \subset \gamma, \ |\eta| < \infty} G(\eta)$$

defines a new function $KG : \Gamma \longrightarrow \mathbb{R}$ for appropriate $G : \Gamma_0 \longrightarrow \mathbb{R}$. The inverse mapping is given by

$$(K^{-1}F)(\eta) = \sum_{\xi \subset \eta} (-1)^{|\eta \setminus \xi|} F(\xi).$$

$\mathscr{B}_c(\mathbb{R}^d)$ denotes the set of all Borel sets with compact closure. In [14] it was shown that the $K-$transform is bijective between the space of all polynomially bounded cylindrical functions F, i.e. $F(\gamma) = F(\gamma_\Lambda)$ for some $\Lambda \in \mathscr{B}_c(\mathbb{R}^d)$, and $B_{bs}(\Gamma_0)$. Where $G \in B_{bs}(\Gamma_0)$ is a bounded function with bounded support, so there exists $N \in \mathbb{N}$ and $\Lambda \in \mathscr{B}_c(\mathbb{R}^d)$ such that

$$G(\eta) = 0, \quad \forall \eta \notin \bigsqcup_{n=0}^{N} \Gamma_{0,\Lambda}^{(n)}$$

with

$$\Gamma_{0,\Lambda}^{(n)} = \{\eta \in \Gamma_0 : \quad \eta \subset \Lambda, \quad |\eta| = n\}.$$

Also further properties of the $K-$transform were studied in [14]. For a measurable function $f : \mathbb{R}^d \longrightarrow \mathbb{R}$ denote by

$$e_\lambda(f,\eta) = \prod_{x\in\eta} f(x), \quad e_\lambda(f,\emptyset) = 1, \quad \eta \in \Gamma_0\backslash\{\emptyset\}$$

the Lebesgue exponential $e_\lambda(f)$. The general scheme and all necessary calculations for dynamics on configuration spaces can be found in [10] and references therein. Given a Markov pre-generator L the dynamics are described by the Kolmogorov equation

$$\frac{\partial F_t}{\partial t} = LF_t.$$

The pairing $\langle F, \mu\rangle = \int_\Gamma F(\gamma)\mathrm{d}\mu(\gamma)$ for $F : \Gamma \longrightarrow \mathbb{R}$ and a probability measure $\mu \in \mathscr{M}^1(\Gamma)$ allows to consider the dual equation for measures

$$\frac{\partial \mu_t}{\partial t} = L^*\mu_t.$$

We construe each probability measure μ_t as a state of the system at time t. So the time evolution is given by $(\mu_t)_{t\geq 0}$. Unfortunately this equation is difficult to handle. Using the K-Transform it is possible to look at the evolutional equation for quasi-observables

$$\frac{\partial G_t}{\partial t} = \hat{L}G_t \tag{14}$$

with $\hat{L} = K^{-1}LK$ on some set of functions $G : \Gamma_0 \longrightarrow \mathbb{R}$, i.e. $B_{bs}(\Gamma_0)$. Given a probability measure μ on Γ the $K-$transform allows to define the correlation measure ρ_μ on Γ_0 via the identity

$$\int_\Gamma (KG)(\gamma)\mathrm{d}\mu(\gamma) = \int_{\Gamma_0} G(\eta)\mathrm{d}\rho_\mu(\eta), \quad G \in B_{bs}(\Gamma_0).$$

Under some general conditions there exist a one to one correspondence between measures on Γ and correlation measures, cf. [14]. If ρ_μ is absolutely continuous with respect to the Lebesgue-Poisson measure $\mathrm{d}\lambda$ then one defines the correlation function as the Radon-Nikodym derivative $k_\mu = \frac{\mathrm{d}\rho_\mu}{\mathrm{d}\lambda}$. Assuming that the evolution μ_t has this property $\rho_{\mu_t} = k_{\mu_t}\mathrm{d}\lambda$ then rewriting equation (14) with the use of

$$\int_{\Gamma_0} (\hat{L}G)(\eta)k(\eta)\mathrm{d}\lambda(\eta) = \int_{\Gamma_0} G(\eta)(L^\Delta k)(\eta)\mathrm{d}\lambda(\eta)$$

we arrive at a strong equation for correlation functions $k_t = k_{\mu_t}$

$$\frac{\partial k_t}{\partial t} = L^\Delta k_t. \tag{15}$$

One great simplification is that in the last two equations the functions depend only on finite configurations. Note that (15) is formulated and will be solved in the strong sense. Since it was originally obtained as a dual equation it is possible to consider the weak form and dual evolutions k_t^D, obtained by the strong solution of the equation for quasi-observables (14). This analysis was done, e.g. in [5], but is not the main goal of this work. The first model will give a brief outline on how to realize this approach. But even having the solution to (15) it is not clear whether this k_t is a correlation function, i.e. corresponds to an evolution of states. Some further analysis is required. For calculations the following two formulas will be essential.

Lemma 3.1. *For $H : \Gamma_0 \times \Gamma_0 \times \Gamma_0 \longrightarrow \mathbb{R}$ and $G : \Gamma_0 \times \mathbb{R}^d \longrightarrow \mathbb{R}$ such that the right-hand sides exist for $|G|$ and $|H|$, the following formulas hold:*

$$\int_{\Gamma_0} \sum_{\xi \subset \eta} H(\xi, \eta \backslash \xi, \eta) \mathrm{d}\lambda(\eta) = \int_{\Gamma_0} \int_{\Gamma_0} H(\xi, \eta, \eta \cup \xi) \mathrm{d}\lambda(\xi) \mathrm{d}\lambda(\eta)$$

and

$$\int_{\Gamma_0} \sum_{x \in \eta} G(\eta, x) \mathrm{d}\lambda(\eta) = \int_{\Gamma_0} \int_{\mathbb{R}^d} G(\eta \cup x, x) \mathrm{d}x \mathrm{d}\lambda(\eta).$$

There is another technique which can be used to analyze the time evolution of such continuous interacting particle systems. This approach is based on generating functionals. All details and proofs for this approach can be found in [15] and [11]. For a given state μ on Γ one can define the so-called Bogoliubov generating functional by

$$B_\mu(\Theta) = \int_{\Gamma} \prod_{x \in \gamma} (1 + \Theta(x)) \mathrm{d}\mu(\gamma),$$

provided that the right-hand side exists. Of course the domain of those Θ for which $B_\mu(\Theta)$ is well-defined depends on μ itself. The Bogoliubov generating functional allows to study properties of μ or even the time evolution via functional analytic methods. Assuming μ has finite local exponential moments, i.e.

$$\int_{\Gamma} e^{\alpha |\gamma_\Lambda|} \mathrm{d}\mu(\gamma) < \infty, \quad \forall \alpha > 0, \quad \forall \Lambda \in \mathscr{B}_c(\mathbb{R}^d)$$

then the generating functional exists for all bounded functions Θ with compact support. According to general results on configuration spaces there is a connection to the correlation measure ρ_μ given by

$$B_\mu(\Theta) = \int_{\Gamma} (K e_\lambda(\Theta))(\gamma) \mathrm{d}\mu(\gamma) = \int_{\Gamma_0} e_\lambda(\Theta, \eta) \mathrm{d}\rho_\mu(\eta).$$

If the correlation measure is absolutely continuous with respect to the Lebesgue-

Poisson measure we can write

$$\begin{aligned} B_\mu(\Theta) &= \int\limits_{\Gamma_0} e_\lambda(\Theta,\eta)k_\mu(\eta)\mathrm{d}\lambda(\eta) \\ &= \sum_{n=0}^{\infty} \frac{1}{n!} \int\limits_{(\mathbb{R}^d)^n} \Theta(x_1)\cdots\Theta(x_n)k^{(n)}(x_1,\dots,x_n)\mathrm{d}x_1\dots\mathrm{d}x_n \end{aligned}$$

with symmetric functions $k^{(n)} : (\mathbb{R}^d)^n \longrightarrow \mathbb{R}_+$ given by

$$k^{(n)}(x_1,\dots,x_n) = \begin{cases} k_\mu(\{x_1,\dots,x_n\}) & ,|\{x_1,\dots,x_n\}| = n \\ 0 & ,|\{x_1,\dots,x_n\}| < n \end{cases}.$$

For $\mu = \pi_z$ one has $k_\mu(\eta) = z^{|\eta|}$ and hence

$$\begin{aligned} B_\mu(\Theta) &= \int\limits_{\Gamma_0} e_\lambda(z\Theta,\eta)\mathrm{d}\lambda(\eta) = \exp\Big(z\int\limits_{\mathbb{R}^d}\Theta(x)\mathrm{d}x\Big) \\ &= \sum_{n=0}^{\infty}\frac{z^n}{n!}\int\limits_{(\mathbb{R}^d)^n}\Theta(x_1)\dots\Theta(x_n)\mathrm{d}x_1\dots\mathrm{d}x_n \end{aligned}$$

for $z \geq 0$. If a functional B admits an such a series expansion it is called entire. In this approach we will be dealing entire generating functionals. As a reminder we give the exact definition of an entire functional.

Definition 3.1. A functional $B : L^1(\mathbb{R}^d,\mathbb{C}) \longrightarrow \mathbb{C}$ is called entire if B is locally bounded and for all $\Theta_0,\Theta \in L^1$ the mapping

$$\mathbb{C} \ni z \longmapsto B(\Theta_0 + z\Theta)$$

is entire. Consequently for each $\Theta_0 \in L^1$ it admits a representation

$$B(\Theta_0 + z\Theta) = \sum_{n=0}^{\infty}\frac{z^n}{n!}d^nB(\Theta_0;\Theta,\dots,\Theta)$$

for $z \in \mathbb{C}$ and $\Theta \in L^1$, where $d^nB(\Theta_0,\cdot)$ is a symmetric bounded n-linear form.

In L^1 spaces it is possible to represent the differentials d^nB by symmetric kernels $\delta^nB \in L^\infty$. Note that a similar result does not hold for L^p spaces with $p > 1$. The following result was shown in [15].

Theorem 3.2. *Let B be an entire functional on L^1. Then each differential $d^nB(\Theta_0;\cdot)$ can be represented by a symmetric kernel $\delta^nB(\Theta_0,\cdot) \in L^\infty((\mathbb{R}^d)^n)$ via*

$$d^nB(\Theta_0,\Theta_1,\dots,\Theta_n) = \int\limits_{(\mathbb{R}^d)^n}\delta^nB(\Theta_0,x_1,\dots,x_n)\Theta_1(x_1)\cdots\Theta_n(x_n)\mathrm{d}x_1\dots\mathrm{d}x_n$$

for $\Theta_1,\dots,\Theta_n \in L^1$. Moreover. the operatornorm of $d^nB(\Theta_0,\cdot)$ coincides with the norm of $\delta^nB(\Theta_0,\cdot)$ and

$$\|\delta^nB(\Theta_0,\cdot)\|_{L^\infty((\mathbb{R}^d)^n)} \leq n!\left(\frac{e}{r}\right)^n \sup_{\|\Theta'\|\leq r}|B(\Theta_0+\Theta')|$$

holds. We call an entire functional of bounded type if the right-hand side is finite for each $r > 0$ and $\Theta_0 \in L^1$.

Applying this to configuration spaces and Bogoliubov generating functionals in [15] the authors showed that the correlation measure is absolutely continuous with respect to the Lebesgue-Poisson measure λ.

Theorem 3.3. *Let μ be a probability measure on Γ and B_μ an entire Bogoliubov generating functional (short GF) on L^1. Then the correlation functions k_μ exists and are given for λ-a.a. $\eta \in \Gamma_0$ by*

$$k_\mu(\eta) = \delta^{|\eta|} B_\mu(0;\eta).$$

For an entire GF thus the correlation functions can be interpreted as the Taylor coefficients. Assuming

$$|B_\mu(\Theta)| \leq C \exp\left(\frac{e}{r}\|\Theta\|_{L^1}\right) \tag{16}$$

for $C \geq 0$ and $r > 0$ it follows

$$k_\mu(\eta) \leq C \left(\frac{e}{r}\right)^{|\eta|}$$

for λ-a.a $\eta \in \Gamma_0$. Therefore condition (16) implies the so-called generalized Ruelle bound, which can be used to show the existence of an evolution of states. As it was shown in [15], one can rewrite the equation for correlation functions to a Cauchy Problem

$$\frac{\partial B_t}{\partial t} = \widetilde{L} B_t, \quad B_t|_{t=0} = B_0,$$

which may be solved in some scale of Banach spaces. (16) suggests to consider a scale of Banach spaces of the form

$$\mathbb{B}'_\alpha = \{B : L^1 \longrightarrow \mathbb{C} : \quad B \text{ is entire and } \|B\|_\alpha < \infty \text{ holds}\}, \tag{17}$$

where the norm is given by $\|B\|_\alpha = \sup\limits_{\Theta \in L^1} |B(\Theta)| e^{-\frac{1}{\alpha}\|\Theta\|_{L^1}}$ for $\alpha > 0$. To show how this general approach can be realized we will analyse the Sourgailis and continuous Contact model as one of the simplest birth and death models in the next section.

3.2 Continuous Sourgailis and Contact Model

The continuous Sourgailis model is the simplest model without interaction. It can be described heuristically by two elementary events birth and death. Both events can be described by spaces homogeneous rates $m = m(t)$ and $\kappa = \kappa(t) \geq 0$. Therefore each particle can die with rate m and at each free site a new particle can be born with rate κ. The Markov pre-generator for such model is given by

$$(L(t)F)(\gamma) = m(t) \sum_{x \in \gamma} (F(\gamma \backslash x) - F(\gamma)) + \kappa(t) \int_{\mathbb{R}^d} (F(\gamma \cup x) - F(\gamma)) \mathrm{d}x.$$

The corresponding expression for $\hat{L}(t)$ on quasi-observables is given by

$$(\hat{L}(t)G)(\eta) = -m(t)|\eta|G(\eta) + \kappa(t)\int_{\mathbb{R}^d} G(\eta \cup x)\mathrm{d}x$$

for $G \in B_{bs}(\Gamma_0)$. For correlation functions we likewise achieve

$$(L^{\Delta}(t)k)(\eta) = -m(t)|\eta|k(\eta) + \kappa(t)\sum_{x\in\eta} k(\eta\backslash x)$$

for appropriate k. The case of time independent coefficients was studied in [3]. The author gave an explicit formula for the solution of (15) and studied the long time behavior. More precisely, he has proved that the correlation functions converge to the correlation functions of the invariant state in some proper Banach space. We will now give a short analysis of the corresponding model with the time dependent coefficients $m = m(t) \geq 0$ and $\kappa = \kappa(t) \geq 0$. For this purpose we will always assume that $\overline{m} = \sup_{t\geq 0} m(t)$ is finite and m, κ are continuous on $\mathbb{R}_+ = [0, \infty)$.

Lemma 3.4. *The unique point wise solution of the equation*

$$\frac{\partial k_t}{\partial t} = L^{\Delta}(t)k_t, \quad k_t|_{t=0} = k_0$$

is given by

$$k_t(\eta) = e^{-|\eta|M(t)}\sum_{\xi\subset\eta} H(t)^{|\xi|}k_0(\eta\backslash\xi), \quad \eta \in \Gamma_0 \tag{18}$$

where $M(t) = \int\limits_0^t m(s)\mathrm{d}s$ *and*

$$H(t) = \int_0^t \kappa(s)e^{M(s)}\mathrm{d}s.$$

Define $h_0 = 1$ and h_n recursively by the formula

$$h_n(t) = n\int_0^t \kappa(s)e^{M(s)}h_{n-1}(s)\mathrm{d}s, \quad n \geq 1.$$

Then, using

$$\int_0^t\int_0^{t_1}\cdots\int_0^{t_{n-1}} f'(t_1)\cdots f'(t_n)\mathrm{d}t_n\cdots\mathrm{d}t_1 = \frac{(f(t)-f(0))^n}{n!}$$

for a continuously differentiable function f, one can show that $h_n(t) = H(t)^n$ holds. Taking into account the definition of the convolution $(k_1 * k_2)(\eta) = \sum\limits_{\xi\subset\eta} k_1(\xi)k_2(\eta\backslash\xi)$, formula (18) takes the form

$$k_t(\eta) = e^{-|\eta|M(t)}\left(H(t)^{|\cdot|} * k_0\right)(\eta) = \left(e_\lambda(H(t)e^{-M(t)}) * e_\lambda(e^{-M(t)})k_0\right)(\eta).$$

Uniqueness follows from general results on ordinary differential equations and to show the validity of formula (18) a simple calculation is required, which shall be omitted.

Let $\mathbb{B}_\alpha = L^\infty(\Gamma_0, e^{-\alpha|\cdot|}\mathrm{d}\lambda)$ with the norm $\|k\|_\alpha = \operatorname*{ess\,sup}_{\eta\in\Gamma_0} |k(\eta)|e^{-\alpha|\eta|}$, which means that each $k \in \mathbb{B}_\alpha$ is sub-poissonian, i.e. $|k(\eta)| \leq \|k\|_\alpha e^{\alpha|\eta|}$. Since $e_\lambda\left(H(t)e^{-M(t)}\right)$ is a correlation function corresponding to $\pi_{H(t)e^{-M(t)}}$ and by Lemma 3.9 from [3] also $e_\lambda\left(e^{-M(t)}\right)k_0$ is a correlation function for $k_0 \in \mathbb{B}_\alpha$ for $k_0 \in \mathbb{B}_\alpha$ we obtain that the convolution k_t is a correlation function, so formula (18) defines an evolution of states μ_t. Fix some $k_0 \in \mathbb{B}_\alpha$ and assume for this section $\kappa(t) \leq zm(t)$ for $t \geq 0$ and some constant $z \geq 0$. Then we have

$$h_n(t) = H(t)^n = \left(\int_0^n \kappa(s)e^{M(s)}\mathrm{d}s\right)^n \leq z^n\left(\int_0^t m(s)e^{M(s)}\mathrm{d}s\right)^n = z^n(e^{M(t)}-1)^n.$$

Hence

$$\begin{aligned}
|k_t(\eta)| &\leq e^{-|\eta|M(t)}\sum_{\xi\subset\eta}(z(e^{M(t)}-1))^{|\xi|}|k_0(\eta\backslash\xi)| \\
&\leq \|k_0\|_\alpha e^{-|\eta|M(t)}\sum_{\xi\subset\eta}(z(e^{M(t)}-1))^{|\xi|}e^{\alpha|\eta\backslash\xi|} \\
&= \|k_0\|_\alpha e^{-|\eta|M(t)}(z(e^{M(t)}-1)+e^\alpha)^{|\eta|} \\
&= \|k_0\|_\alpha(z(1-e^{-M(t)})+e^\alpha e^{-M(t)})^{|\eta|} \\
&\leq \max\{z, e^\alpha\}^{|\eta|}\|k_0\|_\alpha.
\end{aligned}$$

For $e^\alpha \geq z$ we obtain $|k_t(\eta)| \leq \|k_0\|_\alpha e^{\alpha|\eta|}$ and so $k_t \in \mathbb{B}_\alpha$ with $\|k_t\|_\alpha \leq \|k_0\|_\alpha$. Therefore we have shown that for large α the evolution stays in one Banach space. In the next step we will show the continuity of $t \longmapsto k_t \in \mathbb{B}_\alpha$.

Lemma 3.5. *Let α' be arbitrary and fixed. Suppose, that*

$$z \leq e^{\alpha'}. \tag{19}$$

Then for any $\alpha \in \mathbb{R}$ such that

$$\log(2) + \alpha' < \alpha. \tag{20}$$

the mapping

$$\mathbb{R}_+ \ni t \longmapsto k_t \in \mathbb{B}_{\alpha'} \subset \mathbb{B}_\alpha$$

is continuous on $\mathbb{B}_\alpha$ for $k_0 \in \mathbb{B}_{\alpha'}$.

Proof. Let $t, t_0 \in \mathbb{R}_+$. Denote by $t^* = \max\{t, t_0\}$ and $t_* = \min\{t, t_0\}$. Then, for $\xi \subset \eta$ using

$$H(t)^n = h_n(t) \leq z^n(e^{M(t)}-1)^n \leq z^n e^{nM(t)}$$

the following holds

$$\begin{aligned}
|e^{-|\eta|M(t)} - e^{-|\eta|M(t_0)}|h_{|\xi|}(t) &\leq z^{|\xi|}e^{|\xi|M(t)}|\eta|e^{-|\eta|M(t_*)}|M(t)-M(t_0)| \\
&\leq z^{|\xi|}|\eta||M(t)-M(t_0)|e^{\overline{m}|\eta||t-t_0|}.
\end{aligned}$$

Hence, for $n \in \mathbb{N}$

$$|h_n(t) - h_n(t_0)| = n \int_{t_*}^{t^*} \kappa(s) e^{M(s)} h_{n-1}(s) \mathrm{d}s \leq n \int_{t_*}^{t^*} \kappa(s) e^{M(s)} z^{n-1} e^{(n-1)M(s)} \mathrm{d}s.$$

Using $\kappa(s) \leq zm(s)$ the latter expression can be estimated by

$$\begin{aligned}
& nz^n \int_{t_*}^{t^*} \left(\frac{\mathrm{d}}{\mathrm{d}s} e^{M(s)} \right) e^{(n-1)M(s)} \mathrm{d}s \\
= \ & nz^n \left(e^{nM(t^*)} - e^{nM(t_*)} - \frac{n-1}{n} \int_{t_*}^{t^*} nm(s) e^{nM(s)} \mathrm{d}s \right) \\
= \ & nz^n \left(e^{nM(t^*)} - e^{nM(t_*)} - \frac{n-1}{n} \left(e^{nM(t^*)} - e^{nM(t_*)} \right) \right) \\
= \ & z^n (e^{nM(t^*)} - e^{nM(t_*)}) = z^n |e^{nM(t)} - e^{nM(t_0)}|.
\end{aligned}$$

For $a, b > 0$ we use the inequality

$$|b^n - a^n| \leq n|b-a| \max\{a, b\}^{n-1}$$

to obtain

$$\begin{aligned}
& e^{-|\eta| M(t_0)} |h_{|\xi|}(t) - h_{|\xi|}(t_0)| \\
\leq \ & z^{|\xi|} e^{-|\eta| M(t_0)} |e^{|\xi| M(t)} - e^{|\xi| M(t_0)}| \\
\leq \ & z^{|\xi|} e^{-|\eta| M(t_0)} |\xi| \left| e^{M(t)} - e^{M(t_0)} \right| \max \left\{ e^{M(t)}, e^{M(t_0)} \right\}^{|\xi|-1} \\
\leq \ & z^{|\xi|} |\eta| \left| e^{M(t)} - e^{M(t_0)} \right| e^{|\eta|(M(t^*) - M(t_0))} \\
\leq \ & z^{|\xi|} |\eta| \left| e^{M(t)} - e^{M(t_0)} \right| e^{\overline{m}|\eta||t-t_0|}.
\end{aligned}$$

Therefore we have

$$\begin{aligned}
& |k_t(\eta) - k_{t_0}(\eta)| \\
\leq \ & \sum_{\xi \subset \eta} \left| e^{-|\eta| M(t)} h_{|\xi|}(t) - e^{-|\eta| M(t_0)} h_{|\xi|}(t_0) \right| |k_0(\eta \backslash \xi)| \\
\leq \ & \sum_{\xi \subset \eta} h_{|\xi|}(t) |e^{-|\eta| M(t)} - e^{-|\eta| M(t_0)}| |k_0(\eta \backslash \xi)| \\
& + \sum_{\xi \subset \eta} e^{-|\eta| M(t_0)} |h_{|\xi|}(t) - h_{|\xi|}(t_0)| |k_0(\eta \backslash \xi)| \\
\leq \ & \|k_0\|_{\alpha'} |\eta| e^{\overline{m}|\eta||t-t_0|} |M(t) - M(t_0)| \sum_{\xi \subset \eta} z^{|\xi|} e^{\alpha' |\eta \backslash \xi|} \\
& + \|k_0\|_{\alpha'} |\eta| \left| e^{M(t)} - e^{M(t_0)} \right| e^{\overline{m}|\eta||t-t_0|} \sum_{\xi \subset \eta} z^{|\xi|} e^{\alpha' |\eta \backslash \xi|} \\
\leq \ & \|k_0\|_{\alpha'} \left(|M(t) - M(t_0)| + \left| e^{M(t)} - e^{M(t_0)} \right| \right) |\eta| e^{\overline{m}|\eta||t-t_0|} \left(z + e^{\alpha'} \right)^{|\eta|}.
\end{aligned}$$

Now let $\varepsilon > 0$ and take $\delta > 0$ such that for $|t - t_0| < \delta$

$$|M(t) - M(t_0)| + \left|e^{M(t)} - e^{M(t_0)}\right| < \varepsilon$$

and

$$\log(2) + \overline{m}\delta + \alpha' < \alpha$$

holds. According to (19) and (20) we have

$$e^{\overline{m}\delta - \alpha}(z + e^{\alpha'}) \leq 2e^{\overline{m}\delta\alpha' - \alpha} < 2e^{\alpha - \log(2) - \alpha} = 1, \tag{21}$$

which implies $\|k_t - k_{t_0}\|_\alpha \leq Const \cdot \varepsilon \|k_0\|_{\alpha'}$ and thus the desired result. □

Remark 3.1.

1. It is enough to have the strict inequality for either (19) or (20), cf. (21).
2. This proof also shows that for $\xi \subset \eta \in \Gamma_0$

$$e^{-|\eta| M(t)} |h_{|\xi|}(t) - h_{|\xi|}(s)| \leq z^{|\xi|} |\eta| \left| e^{M(t)} - e^{M(s)} \right| e^{|\eta|\overline{m}|t-s|}. \tag{22}$$

We saw that continuity of the solution requires additional regularity, which is reflected by the condition $\alpha - \alpha' > \log(2)$. The reason for such difficulties is due to the fact that we deal with L^∞ spaces. In more general models similar conditions were already used, cf. [5, 8]. To show differentiability we will likewise require regularity of initial date, i.e. $\alpha - \alpha' > \log(2) + \overline{m}$. The precise formulation is the content of the next lemma.

Lemma 3.6. *For $k_0 \in \mathbb{B}_{\alpha'}$ and* (19) *the mapping*

$$\mathbb{R}_+ \ni t \longmapsto k_t \in \mathbb{B}_\alpha$$

is continuously differentiable under the condition

$$\overline{m} + \log(2) + \alpha' < \alpha \tag{23}$$

for $t \geq 0$.

Proof. Using the notation $h_{-1}(t) = 0$ we have for each $\eta \in \Gamma_0$

$$\begin{aligned}
& L^\Delta(t) k_t(\eta) \\
= \; & -|\eta| m(t) k_t(\eta) + \kappa(t) \sum_{x \in \eta} k_t(\eta \backslash x) \\
= \; & -|\eta| m(t) e^{-|\eta| M(t)} \sum_{\xi \subset \eta} h_{|\xi|}(t) k_0(\eta \backslash \xi) \\
& + \kappa(t) \sum_{x \in \eta} \sum_{\xi \subset (\eta \backslash x)} e^{-|\eta| M(t)} e^{M(t)} h_{|\xi|}(t) k_0(\eta \backslash (\xi \cup x)) \\
= \; & -|\eta| m(t) e^{-|\eta| M(t)} \sum_{\xi \subset \eta} h_{|\xi|}(t) k_0(\eta \backslash \xi) \\
& + \kappa(t) e^{M(t)} \sum_{\xi \subset \eta} \sum_{x \in \xi} e^{-|\eta| M(t)} h_{|\xi| - 1}(t) k_0(\eta \backslash \xi) \\
= \; & \sum_{\xi \subset \eta} k_0(\eta \backslash \xi) \left(-|\eta| m(t) e^{-|\eta| M(t)} h_{|\xi|}(t) + \kappa(t) e^{M(t)} |\xi| e^{-|\eta| M(t)} h_{|\xi| - 1}(t) \right).
\end{aligned}$$

Similar calculations show for $h \in \mathbb{R}$ such that $t+h, t \geq 0$

$$\begin{aligned}
&\frac{k_{t+h}(\eta) - k_t(\eta)}{h} \\
= &\frac{1}{h}\left(e^{-|\eta| M(t+h)} \sum_{\xi \subset \eta} h_{|\xi|}(t+h) k_0(\eta \backslash \xi) - e^{-|\eta| M(t)} \sum_{\xi \subset \eta} h_{|\xi|}(t) k_0(\eta \backslash \xi) \right) \\
= &\sum_{\xi \subset \eta} k_0(\eta \backslash \xi) \left(\frac{e^{-|\eta| M(t+h)} h_{|\xi|}(t+h) - e^{-|\eta| M(t)} h_{|\xi|}(t)}{h} \right) \\
= &\sum_{\xi \subset \eta} k_0(\eta \backslash \xi) \left(h_{|\xi|}(t+h) \frac{e^{-|\eta| M(t+h)} - e^{-|\eta| M(t)}}{h} + e^{-|\eta| M(t)} \frac{h_{|\xi|}(t+h) - h_{|\xi|}(t)}{h} \right).
\end{aligned}$$

The difference $\dfrac{k_{t+h}(\eta) - k_t(\eta)}{h} - L^{\Delta}(t) k_t(\eta)$ has now the form

$$\begin{aligned}
&\sum_{\xi \subset \eta} k_0(\eta \backslash \xi) \left(h_{|\xi|}(t+h) \frac{e^{-|\eta| M(t+h)} - e^{-|\eta| M(t)}}{h} + |\eta| m(t) e^{-|\eta| M(t)} h_{|\xi|}(t) \right) \\
+ &\sum_{\xi \subset \eta} k_0(\eta \backslash \xi) \left(e^{-|\eta| M(t)} \frac{h_{|\xi|}(t+h) - h_{|\xi|}(t)}{h} - \kappa(t) e^{M(t)} |\xi| e^{-|\eta| M(t)} h_{|\xi|-1}(t) \right)
\end{aligned}$$

and the multiplicant in the first summand can be rewritten to

$$\begin{aligned}
&h_{|\xi|}(t+h) \frac{e^{-|\eta| M(t+h)} - e^{-|\eta| M(t)}}{h} + |\eta| m(t) e^{-|\eta| M(t)} h_{|\xi|}(t) \\
= \; &h_{|\xi|}(t+h) \left(\frac{e^{-|\eta| M(t+h)} - e^{-|\eta| M(t)}}{h} + |\eta| m(t) e^{-|\eta| M(t)} \right) \\
&+ |\eta| m(t) e^{-|\eta| M(t)} (h_{|\xi|}(t) - h_{|\xi|}(t+h)).
\end{aligned}$$

Now let $\varepsilon > 0$ and take $\min\{\varepsilon, 1\} > \delta > 0$ such that

1. $\left| m(t) - \dfrac{M(t+h) - M(t)}{h} \right| < \varepsilon$
2. $\left| \kappa(s) e^{M(s)} - \kappa(t) e^{M(t)} \right| < \varepsilon$
3. $\left| e^{M(s)} - e^{M(t)} \right| < \varepsilon$
4. $(1+\delta)\overline{m} + \log(2) + \alpha' < \alpha$

holds for $|t-s| < |h| < \delta$. Then we obtain by (22) for such h

$$|\eta| m(t) e^{-|\eta| M(t)} |h_{|\xi|}(t) - h_{|\xi|}(t+h)| \leq |\eta|^2 m(t) z^{|\xi|} \varepsilon e^{|\eta| \overline{m} \delta}.$$

The first part can be estimated by

$$\begin{aligned}
& h_{|\xi|}(t+h)\left|\frac{e^{-|\eta|M(t+h)}-e^{-|\eta|M(t)}}{h}+|\eta|m(t)e^{-|\eta|M(t)}\right| \\
\leq\ & z^{|\xi|}e^{|\xi|M(t+h)}e^{-|\eta|M(t)}\left|m(t)|\eta|+\frac{e^{-|\eta|M(t+h)+|\eta|M(t)}-1}{h}\right| \\
\leq\ & z^{|\xi|}e^{|\eta|\overline{m}\delta}|\eta|\left|m(t)-\frac{M(t+h)-M(t)}{h}\right| \\
& +\frac{z^{|\xi|}e^{|\eta|\overline{m}\delta}}{|h|}\sum_{k=2}^{\infty}\frac{|\eta|^k}{k!}|M(t+h)-M(t)|^k \\
\leq\ & z^{|\xi|}e^{|\eta|\overline{m}\delta}|\eta|\varepsilon+z^{|\xi|}e^{|\eta|\overline{m}\delta}|h|\sum_{k=2}^{\infty}\frac{|\eta|^k|h|^{k-2}\overline{m}^k}{k!} \\
\leq\ & z^{|\xi|}e^{|\eta|\overline{m}\delta}\left(|\eta|+\sum_{k=2}^{\infty}\frac{|\eta|^k\overline{m}^k}{k!}\right)\varepsilon \\
\leq\ & z^{|\xi|}e^{|\eta|\overline{m}\delta}\left(|\eta|+e^{|\eta|\overline{m}}\right)\varepsilon.
\end{aligned}$$

Altogether we have shown

$$h_{|\xi|}(t+h)\left|\frac{e^{-|\eta|M(t+h)}-e^{-|\eta|M(t)}}{h}+m(t)|\eta|e^{-|\eta|M(t)}\right|\leq z^{|\xi|}e^{|\eta|\overline{m}\delta}\left(|\eta|+e^{|\eta|\overline{m}}\right)\varepsilon.$$

Taking now the sum the first part of the difference $\frac{k_{t+h}(\eta)-k_t(\eta)}{h}-L^{\Delta}(t)k_t(\eta)$ can be estimated by

$$\begin{aligned}
& \sum_{\xi\subset\eta}|k_0(\eta\backslash\xi)|\left|h_{|\xi|}(t+h)\frac{e^{-|\eta|M(t+h)}-e^{-|\eta|M(t)}}{h}+|\eta|m(t)e^{-|\eta|M(t)}h_{|\xi|}(t)\right| \\
\leq\ & \|k_0\|_{\alpha'}\varepsilon\left(|\eta|^2m(t)e^{|\eta|\overline{m}\delta}+e^{|\eta|\overline{m}\delta}(|\eta|+e^{\overline{m}|\eta|})\right)\sum_{\xi\subset\eta}z^{|\xi|}e^{\alpha'|\eta\backslash\xi|} \\
=\ & \|k_0\|_{\alpha'}\varepsilon\left(|\eta|^2m(t)+|\eta|+e^{\overline{m}|\eta|}\right)e^{|\eta|\overline{m}\delta}\left(z+e^{\alpha'}\right)^{|\eta|} \\
\leq\ & \|k_0\|_{\alpha'}e^{\alpha|\eta|}\varepsilon\left(|\eta|^2m(t)+|\eta|+1\right)e^{(1+\delta)\overline{m}|\eta|-\alpha|\eta|}\left(z+e^{\alpha'}\right)^{|\eta|}.
\end{aligned}$$

Using $(1+\delta)\overline{m}+\log(2)+\alpha'<\alpha$ we consequently obtain

$$e^{(1+\delta)\overline{m}-\alpha}\left(z+e^{\alpha'}\right)\leq 2e^{(1+\delta)\overline{m}+\alpha'-\alpha}<1$$

and thus it implies for $\beta\geq 0$

$$|\eta|^{\beta}e^{((1+\delta)\overline{m}-\alpha)|\eta|}\left(z+e^{\alpha'}\right)^{|\eta|}\leq Const$$

pointwise, which gives the desired result. In the same way we estimate the second difference with $t_* = \min(t, t+h)$ and $t^* = \max(t, t+h)$

$$\begin{aligned} & \left| e^{-|\eta|M(t)} \frac{h_{|\xi|}(t+h) - h_{|\xi|}(t)}{h} - \kappa(t) e^{M(t)} |\xi| e^{-|\eta|M(t)} h_{|\xi|-1}(t) \right| \\ = \ & |\xi| e^{-|\eta|M(t)} \left| \frac{1}{h} \int_t^{t+h} \kappa(s) e^{M(s)} h_{|\xi|-1}(s) \mathrm{d}s - \kappa(t) e^{M(t)} h_{|\xi|-1}(t) \right| \\ \leq \ & e^{-|\eta|M(t)} \frac{|\xi|}{|h|} \int_{t_*}^{t^*} |\kappa(s) e^{M(s)} h_{|\xi|-1}(s) - \kappa(t) e^{M(t)} h_{|\xi|-1}(t)| \mathrm{d}s. \end{aligned}$$

The integrand can be estimated by

$$\begin{aligned} & \left| \kappa(s) e^{M(s)} h_{|\xi|-1}(s) - \kappa(t) e^{M(t)} h_{|\xi|-1}(t) \right| \\ \leq \ & \kappa(s) e^{M(s)} \left| h_{|\xi|-1}(s) - h_{|\xi|-1}(t) \right| + h_{|\xi|-1}(t) \left| \kappa(s) e^{M(s)} - \kappa(t) e^{M(t)} \right| \\ \leq \ & \overline{\kappa} e^{\overline{m}(t+\delta)} |\xi| z^{|\xi|-1} \left| e^{M(t)} - e^{M(s)} \right| e^{|\eta|\overline{m}\delta} + \varepsilon z^{|\xi|-1} e^{|\xi|M(t)} \\ \leq \ & z^{|\xi|} e^{|\xi|M(t)} \varepsilon \left(\frac{\overline{\kappa} e^{\overline{m}(t+1)}}{z} |\eta| e^{|\eta|\overline{m}\delta} + z^{-1} \right) \\ \leq \ & z^{|\xi|} e^{|\eta|M(t)} \varepsilon \left(\frac{\overline{\kappa} e^{\overline{m}(t+1)}}{z} |\eta| e^{|\eta|\overline{m}\delta} + z^{-1} \right) \end{aligned}$$

with $\overline{\kappa} = \sup\limits_{t \geq 0} \kappa(t)$ and thus

$$\begin{aligned} & e^{-|\eta|M(t)} \frac{|\xi|}{|h|} \int_{t_*}^{t^*} |\kappa(s) e^{M(s)} h_{|\xi|-1}(s) - \kappa(t) e^{M(t)} h_{|\xi|-1}(t)| \mathrm{d}s \\ \leq \ & e^{-|\eta|M(t)} |\xi| z^{|\xi|} e^{|\eta|M(t)} \varepsilon \left(\frac{\overline{\kappa} e^{\overline{m}(t+1)}}{z} |\eta| e^{|\eta|\overline{m}\delta} + z^{-1} \right) \\ \leq \ & z^{|\xi|} \varepsilon \left(\frac{\overline{\kappa} e^{\overline{m}(t+1)}}{z} |\eta|^2 e^{|\eta|\overline{m}\delta} + z^{-1} |\eta| \right) \end{aligned}$$

Now taking the sum $\sum\limits_{\xi \subset \eta}$ we obtain the assertion analogous to the previous difference. □

We are interested in solutions on some Banach spaces $\mathbb{B}_\alpha$. Right now we have a pointwise solution formula, and under some restrictions, continuity and differentiability properties for some initial values. It still remains to find some Banach space such that the solution formula defines a continuous operator, which is differentiable in some norm on some subspace. From Lemma 3.7 it is natural to consider this in the norm $\| \cdot \|_\alpha$ together with some closed subspace.

Theorem 3.7. *For each $\alpha' < \alpha$ with $z \leq e^{\alpha'}$ and $\overline{m} + \log(2) + \alpha' < \alpha$ there exists a family of contraction operators $(T^\Delta_{\alpha'\alpha}(t))_{t \geq 0}$ on $\mathbb{B} := \overline{\mathbb{B}_{\alpha'}}^{\|\cdot\|_\alpha}$ with the properties*

1. *$T^{\Delta}_{\alpha'\alpha}(t)$ is strongly continuous on $\mathbb{B}$*
2. *$[0, T(\alpha, \alpha')) \ni t \longmapsto T^{\Delta}_{\alpha'\alpha}(t)k \in \mathbb{B}$ is continuously differentiable for $k \in \mathbb{B}_{\alpha'}$ with the derivative*

$$\frac{\mathrm{d}T^{\Delta}_{\alpha'\alpha}(t)k}{\mathrm{d}t} = L^{\Delta}(t)T^{\Delta}_{\alpha'\alpha}(t)k$$

on $\mathbb{B}$.

Hence for $k_0 \in \mathbb{B}_{\alpha'}$ the unique solution of the Cauchy problem

$$\frac{\partial k_t}{\partial t} = L^{\Delta}(t)k_t, \quad k_t|_{t=0} = k_0$$

on $\mathbb{B}$ is given by $k_t = T^{\Delta}_{\alpha'\alpha}(t)k_0$ and moreover $k_t \in \mathbb{B}_{\alpha'} \subset \mathbb{B}_{\alpha}$.

Note that the family $(T^{\Delta}_{\alpha'\alpha}(t))_{t\geq 0}$ is not a semigroup. Under slight changes it is possible to give, at least, a heuristic formula for an evolution family $U^{\Delta}_{\alpha'\alpha}(t, s)$.

Proof. We have shown $\|k_t\|_{\alpha} \leq \|k_0\|_{\alpha}$ for $k_0 \in \mathbb{B}_{\alpha'} \subset \mathbb{B}_{\alpha}$. Hence the densely defined operator $T^{\Delta}_{\alpha'\alpha}(t)k_0 = k_t$ has a unique extension on $\mathbb{B}$, which we denote again by $T^{\Delta}_{\alpha'\alpha}(t)$. Strong continuity follows from the contraction property and Lemma 3.5. Strong differentiability was shown in Lemma 3.7 and therefore for each $k_0 \in \mathbb{B}_{\alpha'}$ there exists a solution given by $k_t = T^{\Delta}_{\alpha'\alpha}(t)k_0 \in \mathbb{B}_{\alpha'} \subset \mathbb{B}$. The uniqueness follows from the uniqueness of the pointwise solution formula. □

Having the existence of an evolution we will discuss some conditions for invariant states and convergence to invariant states. One special case is the time independent dynamics.

Remark 3.2. Assume that m is not integrable, i.e. $M(t) \to \infty$ for $t \to \infty$, e.g. if m is periodic.

1. For some initial condition $k_0 \in \mathbb{B}_{\alpha'}$ one has the solution

$$k_t(\eta) = e^{-|\eta|M(t)} \sum_{\xi \subset \eta} H(t)^{|\xi|} k_0(\eta \backslash \xi).$$

In the special case $k_0(\eta) = e^{\alpha'|\eta|}$ we obtain

$$k_t(\eta) = e^{-|\eta|M(t)} \left(H(t) + e^{\alpha'}\right)^{|\eta|} = \left(H(t)e^{-M(t)} + e^{\alpha'}e^{-M(t)}\right)^{|\eta|}.$$

Using $\underline{\kappa} = \min_{t\geq 0} \kappa(t)$ we obtain

$$\underline{\kappa}t \leq H(t) \leq z\left(e^{M(t)} - 1\right)$$

and hence

$$\left(\underline{\kappa}te^{-M(t)} + e^{\alpha'}e^{-M(t)}\right)^{|\eta|} \leq k_t(\eta) \leq \left(z + (e^{\alpha'} - z)e^{-M(t)}\right)^{|\eta|}.$$

For a general initial condition $k_0 \in \mathbb{B}_{\alpha'}$ we obtain by $H(t)e^{-M(t)} \leq z$ using the decomposition

$$\begin{aligned} k_t(\eta) &= e^{-|\eta|M(t)}k_0(\eta) + H(t)^{|\eta|}e^{-|\eta|M(t)} \\ &+ \sum_{\xi\subset\eta,\xi\neq\emptyset,\xi\neq\eta} H(t)^{|\xi|}e^{-|\xi|M(t)}k_0(\eta\backslash\xi)e^{-|\eta\backslash\xi|M(t)} \end{aligned}$$

that the existence of the limit $\lim\limits_{t\to\infty} k_t(\eta) = k(\eta)$ is equivalent to the existence of the limit $\lim\limits_{t\to\infty} H(t)e^{-M(t)} = a$ and we have

$$k(\eta) = \lim_{t\to\infty} k_t(\eta) = a^{|\eta|}$$

for which $0 \leq a \leq z$ holds. So the condition $\kappa(t) \leq zm(t)$ and $k_0 \in \mathbb{B}_{\alpha'}$ for some $\alpha' \in \mathbb{R}$ imply that the limiting state will be always Poissonian, i.e. π_a.

2. Now take $k_0(\eta) = e^{\alpha'|\eta|}$ for some $\alpha' \in \mathbb{R}$ and assume that the limit $\lim\limits_{t\to\infty} H(t)e^{-M(t)} = a$ exists. Then for each $\alpha \in \mathbb{R}$, which satisfies $a < e^{\alpha}$, we have $k_t \to e_\lambda(a)$ for $t \to \infty$ in $\mathbb{B}_\alpha$. To show this, let $\varepsilon > 0$ with $a \neq \frac{\varepsilon}{2}$, $a + \frac{\varepsilon}{2} < e^{\alpha}$ and take $t_0 > 0$ such that for each $t \geq t_0$

   (a) $a - \frac{\varepsilon}{2} \leq H(t)e^{-M(t)} \leq a + \frac{\varepsilon}{2}$

   (b) $e^{\alpha'}e^{-M(t)} \leq \frac{\varepsilon}{2}$

   (c) $\left|H(t)e^{-M(t)} + e^{\alpha'}e^{-M(t)} - a\right| \leq \varepsilon$

   holds. Then the assertion follows from

$$\begin{aligned} \left|k_t(\eta) - a^{|\eta|}\right| &\leq |\eta| \frac{\left|H(t)e^{-M(t)} + e^{\alpha'}e^{-M(t)} - a\right|}{\max\left\{a, H(t)e^{-M(t)} + e^{\alpha'}e^{-M(t)}\right\}} \\ &\quad \times \max\left\{a, H(t)e^{-M(t)} + e^{\alpha'}e^{-M(t)}\right\}^{|\eta|} \\ &\leq \varepsilon|\eta| \frac{\max\left\{a, a+\varepsilon\right\}^{|\eta|}}{\max\left\{a, a - \frac{\varepsilon}{2}\right\}} \\ &= \varepsilon \frac{1}{a - \frac{\varepsilon}{2}} e^{\alpha|\eta|}|\eta| \left(e^{-\alpha|\eta|}(a+\varepsilon)\right)^{|\eta|} \\ &\leq Const \cdot \varepsilon e^{\alpha|\eta|} \end{aligned}$$

   for $a \neq 0$. The case $a = 0$ can be shown analogously.

3. The condition $t \longmapsto \dfrac{\kappa(t)}{m(t)}$ is mononically increasing implies

$$H(t) = \int_0^t \frac{\kappa(s)}{m(s)} m(s)e^{M(s)}\mathrm{d}s \leq \frac{\kappa(t)}{m(t)}e^{M(t)}.$$

Hence $\lim\limits_{t\to\infty} \dfrac{\kappa(t)}{m(t)} = z$ and moreover

$$\frac{\mathrm{d}}{\mathrm{d}t} H(t)e^{-M(t)} = \kappa(t) - m(t)H(t)e^{-M(t)} \geq 0$$

implies $\lim\limits_{t\to\infty} H(t)e^{-M(t)} = z$. Consequently we have shown

$$\lim_{t\to\infty} k_t(\eta) = \lim_{t\to\infty} \left(H(t)e^{-M(t)}\right)^{|\eta|} = z^{|\eta|}$$

pointwise for all $\eta \in \Gamma_0$.

4. Now take $z = \dfrac{\kappa}{m}$ time independent. Then π_z is an invariant state and for $k_0(\eta) = e^{\alpha'|\eta|}$ we obtain

$$k_t(\eta) = e^{-|\eta|M(t)} z^{|\xi|}(e^{M(t)} - 1)^{|\xi|} e^{\alpha'|\eta\backslash\xi|} = \left(z + (e^{\alpha'} - z)e^{-M(t)}\right)^{|\eta|}.$$

Therefore the time evolution is Poissonian and converges to the invariant state π_z. We obtain with $\max\{z, e^{\alpha'}\} > 0$

$$|k_t(\eta) - k_{inv}(\eta)| \leq e^{-M(t)} \frac{|z - e^{\alpha'}|}{\max\{z, e^{\alpha'}\}} |\eta| \max\{z, e^{\alpha'}\}^{|\eta|}$$

and hence $k_t \to k_{inv}$ in $\mathbb{B}_\alpha$ with $e^\alpha > \max\{z, e^{\alpha'}\}$.

5. Now consider $m(t) = a > 0$ and $\kappa(t) = e^{-bt}$, then we obtain

$$H(t) = \begin{cases} \dfrac{e^{(a-b)t} - 1}{a - b} & , a \neq b \\ t & , a = b \end{cases}.$$

The expression $H(t)e^{-M(t)} = \dfrac{e^{-bt} - e^{-at}}{a - b}$ converges for $b > 0$ to 0 and hence $k_t(\eta) \to 0^{|\eta|}$, so all particles will die. In the case $b < 0$ the expression k_t does not have a limit for $t \to \infty$.

More generally now let the death rate be space dependent and introduce some branching, meaning that each particle may produce another new particle. This model was already described in the introduction and the Markov pre-generator has for quasi-observables the form

$$\begin{aligned}(\hat{L}(t)G)(\eta) &= -m(t)|\eta|G(\eta) - \left(\sum_{x\in\eta} P_t(x)\right)G(\eta) \\ &\quad + \int_{\mathbb{R}^d} \sum_{y\in\eta} a_t(x-y)G((\eta\backslash y)\cup x)\mathrm{d}x + \int_{\mathbb{R}^d} \sum_{y\in\eta} a_t(x-y)G(\eta\cup x)\mathrm{d}x.\end{aligned}$$

for $G \in B_{bs}(\Gamma_0)$. We consider this model under the assumptions

1. $m \geq 0$ is a continuous function on $[0, T]$ for some $T > 0$

2. $P_t : \mathbb{R}^d \longrightarrow \mathbb{R}_+$ with $P_t(x) = P_t(-x)$ satisfies

$$P_\bullet \in C([0,T]; L^\infty(\mathbb{R}^d))$$

3. $0 \le a_t \in L^1(\mathbb{R}^d) \cap L^\infty(\mathbb{R}^d)$ with $a_t(x) = a_t(-x)$ for $t \in [0,T]$

4. $[0,T] \ni t \longmapsto a_t \in L^p(\mathbb{R}^d)$ is continuous for $p = 1, \infty$.

In such case $\hat{L}(t)$ can be realised as a bounded linear operator $\hat{L}(t) \in L(\mathbb{B}'_\alpha, \mathbb{B}'_{\alpha'})$ for all $\alpha' < \alpha$, where $\mathbb{B}'_\alpha = L^1(\Gamma_0, e^{\alpha|\cdot|}\mathrm{d}\lambda)$ with the norm

$$\|G\|_\alpha = \int_{\Gamma_0} |G(\eta)| e^{\alpha|\eta|} \mathrm{d}\lambda(\eta) = \sum_{n=0}^{\infty} \frac{e^{\alpha n}}{n!} \int_{(\mathbb{R}^d)^n} |G^{(n)}(x_1, \ldots, x_n)| \mathrm{d}x_1 \ldots \mathrm{d}x_n$$

is a scale of Banach spaces of type 2.

Lemma 3.8. *The expression given for $\hat{L}(t)$ defines a bounded linear operator $\hat{L}(t) \in L(\mathbb{B}'_\alpha, \mathbb{B}'_{\alpha'})$ such that the mapping*

$$[0,T] \ni t \longmapsto \hat{L}(t) \in L(\mathbb{B}'_\alpha, \mathbb{B}'_{\alpha'})$$

is continuous in the uniform operator topology for $\alpha' < \alpha$.

Proof. For $\alpha' < \alpha$ it is simple to show

$$\|\hat{L}(t)\|_{\alpha\alpha'} \le \frac{m(t) + \|P_t\|_{L^\infty} + \|a_t\|_{L^1}}{e(\alpha - \alpha')} + \frac{4\|a_t\|_\infty e^{-\alpha'}}{e^2(\alpha - \alpha')^2}, \qquad (24)$$

which shows the first assertion. Since the operator $\hat{L}$ depends linearly on the parameters m, P, a the continuity follows immediately from (24). □

In order to solve the equation for quasi-observables it would be sufficient to show that

$$(A(t)G)(\eta) = \int_{\mathbb{R}^d} \sum_{x \in \eta} a_t(x - y) G(\eta \cup x) \mathrm{d}y$$

generates for each $t \in [0,T]$ a C_0-semigroup such that Theorem 2.1 and 2.3 are applicable. The existence of a C_0-semigroup was proved in [5] for more general dynamics. Therefore we will realise this approach in the section 3.3. Instead we will turn to correlation functions and solve the corresponding equation for the particle densities. For correlation functions the following representation of $L^\Delta(t)$ holds for appropriate G and correlation functions k satisfying $k(\eta) \le |\eta| C^{|\eta|}$, cf. [18],

$$\begin{aligned}(L^\Delta(t)k)(\eta) &= -|\eta| m(t) k(\eta) - \sum_{x \in \eta} P_t(x) k(\eta) \\ &\quad + \sum_{x \in \eta} \sum_{y \in (\eta \backslash x)} k(\eta \backslash x) a_t(x - y) + \sum_{x \in \eta} \int_{\mathbb{R}^d} a_t(x - y) k((\eta \backslash x) \cup y) \mathrm{d}y.\end{aligned}$$

Analogous to previous calculations one can show that $L^{\Delta}(t)$ satisfies the same bound as in (24) and continuity. To analyse the long time behavior of this system we will consider only the first correlation function, which can be construed as a density. For $\eta = \{x\}$ the corresponding equation takes the form

$$\begin{aligned}\frac{\partial k_t^{(1)}(x)}{\partial t} &= -m(t)k_t^{(1)}(x) - P_t(x)k_t(x) + \int\limits_{\mathbb{R}^d} a_t(x-y)k_t^{(1)}(y)\mathrm{d}y \\ &\leq -(\underline{m} + \underline{P}(x))k_t^{(1)}(x) + z\int\limits_{\mathbb{R}^d} a_t(x-y)\mathrm{d}y \\ &= -M(x)k_t^{(1)}(x) + z\kappa(t)\end{aligned}$$

with $\kappa(t) = \int\limits_{\mathbb{R}^d} a_t(y)\mathrm{d}y \geq 0$, $M(x) = \underline{m} + \underline{P}(x)$, $\underline{m} = \inf\limits_{t\geq 0} m(t) \geq 0$, $\underline{P}(x) = \inf\limits_{t\geq 0} P_t(x) \geq 0$ and the assumption $k_t(x) \leq z$. This leads to the bound

$$k_t^{(1)}(x) \leq e^{-M(x)t}k_0(x) + ze^{-M(x)t}\int\limits_0^t \kappa(s)e^{M(x)s}\mathrm{d}s$$

for the solution $k_t^{(1)}(x)$. If κ asymptotically has exponential decay, then clearly $k_t^{(1)}(x) \to 0, \quad t \to \infty$ holds for $M(x) > 0$. Of course our approach and our assumptions have simplified the situation a lot. For more specific properties more detailed analysis is required. In applications one would use computer simulations instead of solving the equations explicitly or at least asymptotically. To show the existence of a solution we will work in the space

$$X_T = C([0,T]; L^{\infty}(\mathbb{R}^d)), \quad \|v\|_T = \sup_{(t,x)\in[0,T]\times\mathbb{R}^d} |v_t(x)| = \sup_{t\in[0,T]} \|v_t\|_{L^{\infty}} \qquad (25)$$

and denote the closed cone of all non-negative functions $v \in X_T$ by X_T^+. For $T' < T$ one has the natural embedding $X_{T'} \subset X_T$, where $X_{T'}$ is a closed subspace.

Lemma 3.9. *Let $A \in X_T^+$, $0 \leq a_t \in L^1(\mathbb{R}^d)$ for $t \in [0,T]$ and assume $(t,x) \longmapsto a_t(x) \geq 0$ is measurable with*

$$\sup_{t\in[0,T]} \int\limits_{\mathbb{R}^d} a_t(x)\mathrm{d}x = \overline{a} < \infty.$$

Then the equation

$$\frac{\partial k_t(x)}{\partial t} = -A(t,x)k_t(x) + (a_t * k_t)(x), \quad k_t|_{t=0} = k_0 \in L^{\infty}(\mathbb{R}^d) \qquad (26)$$

has a unique non-negative solution $k_t \in L^{\infty}(\mathbb{R}^d)$ for $k_0 \geq 0$ and $t \in [0, \widetilde{T})$ with

$$\widetilde{T} = \begin{cases} T & ,\overline{a} = 0, \\ \min\left\{T, \frac{1}{\overline{a}}\right\} & ,\overline{a} > 0 \end{cases}.$$

This solution satisfies $0 \leq k_{\bullet} \in C^1([0,T']; L^{\infty}(\mathbb{R}^d))$ for each $T' < \widetilde{T}$.

Proof. Define the mapping $\Phi : X_{T'} \longrightarrow X_{T'}$ given by

$$(\Phi v)_t(x) = \exp\Bigg(-\int\limits_0^t A(s,x)\mathrm{d}s\Bigg)k_0(x) + \int\limits_0^t \exp\Bigg(-\int\limits_s^t A(\tau,x)\mathrm{d}\tau\Bigg)(a_s * v_s)(x)\mathrm{d}s$$

for $T' < \widetilde{T}$. Clearly Φ is positivity preserving and by

$$|(a_s * v_s)(x)| \leq (a_s * |v_s|)(x) \leq \|a_s\|_{L^1}\|v_s\|_{L^\infty} \leq \overline{a}\|v\|_{T'}$$

we obtain

$$\begin{aligned} |(\Phi v)_t(x)| &\leq k_0(x) + \int\limits_0^t \exp\Bigg(-\int\limits_s^t A(\tau,x)\mathrm{d}\tau\Bigg)|a_s * v_s|(x)\mathrm{d}s \\ &\leq \|k_0\|_{L^\infty} + \int\limits_0^t \overline{a}\|v\|_{T'}\mathrm{d}s \\ &\leq \|k_0\|_{L^\infty} + T'\overline{a}\|v\|_{T'} \end{aligned}$$

and hence $\Phi v \in X_{T'}$ for $v \in X_{T'}$, note that $t \longmapsto (\Phi v)_t \in L^\infty(\mathbb{R}^d)$ is continuous. In the same way

$$|(\Phi v)_t(x) - (\Phi w)_t(x)| \leq \int\limits_0^t \exp\Bigg(-\int\limits_s^t A(\tau,x)\mathrm{d}\tau\Bigg)(a_s * |v_s - w_s|)(x)\mathrm{d}s \leq T'\overline{a}\|v-w\|_{T'}$$

implies that Φ has the contraction property. Thus the sequence $(v^{(n)})_{n\in\mathbb{N}} \subset X_{T'}^+$ given by $v^{(0)} = k_0$ and $v^{(n+1)} = \Phi v^{(n)}$ is a fundamental sequence and hence has a limit $v = \lim\limits_{n\to\infty} v^{(n)} \in X_{T'}^+$. Consequently $v = \Phi v$, i.e.

$$v_t(x) = \exp\Bigg(-\int\limits_0^t A(s,x)\mathrm{d}s\Bigg)k_0(x) + \int\limits_0^t \exp\Bigg(\int\limits_s^t A(\tau,x)\mathrm{d}\tau\Bigg)(a_s * v_s)(x)\mathrm{d}s \quad (27)$$

for a.a. $x \in \mathbb{R}^d$ holds, which shows the existence of a solution to (26). Since every solution of (26) solves (27) the uniqueness follows for $t \in [0,T']$ and hence on $[0,\widetilde{T})$. ☐

Corollary 3.10. *Let $A \in X_T^+$ for each $T > 0$ and $0 \leq a_t \in L^1(\mathbb{R}^d)$ for $t \geq 0$, $(t,x) \longmapsto a_t(x) \geq 0$ be measurable and assume*

$$\sup_{t\geq 0} \int\limits_{\mathbb{R}^d} a_t(x)\mathrm{d}x = \overline{a} < \infty$$

Then the equation

$$\frac{\partial k_t(x)}{\partial t} = -A(t,x)k_t(x) + (a_t * k_t)(x), \quad k_t|_{t=0} = k_0 \in L^\infty(\mathbb{R}^d)$$

has a unique non-negative solution $k_t \in L^\infty(\mathbb{R}^d)$ for $k_0 \geq 0$ and $t \geq 0$. Moreover $k_\bullet \in C^1([0,T]; L^\infty(\mathbb{R}^d))$ holds for each $T > 0$.

Proof. Under this assumption one can take $\widetilde{T} = \dfrac{1}{\overline{a}}$ and hence consider iteratively the same Cauchy problem with initial conditions $k_t|_{t=0} = k_{lT'}$ with $l \in \mathbb{N}$ and $T' < \dfrac{1}{\overline{a}}$. □

In order to apply Lemma 3.11 we need

$$\operatorname*{ess\,sup}_{(t,x)\in[0,T]\times\mathbb{R}^d} m(t) + P_t(x) < \infty$$

and

$$\sup_{t\in[0,T]} \int_{\mathbb{R}^d} a_t(x)\mathrm{d}x = \overline{a} < \infty.$$

Both conditions are satisfyied since $0 \le m \in C([0,T])$, $P_\bullet \in C([0,T]; L^\infty(\mathbb{R}^d))$ and $a_\bullet \in C([0,T]; L^p(\mathbb{R}^d))$ for $p = 1, \infty$. Hence there exists a unique solution to the equation for densities.

3.3 Bolker-Dieckman-Law-Pacala Model

In this section we will discuss an ecological birth and death model. Each individual may die due to a space independent mortality rate m and due to competition of individuals. This competition is described translation invariant by a competition kernel a^-, i.e $a^-(x,y) \equiv a^-(x-y) = a^-(y-x)$. High values for a^- lead to high probabilities of death. Analogously each individual can produce another individual, where the probability distribution of this elementary event is given by the dispersion kernel a^+. Therefore we can describe this model by the following Markov pre-generator

$$(LF)(\gamma) = \sum_{x\in\gamma}(m+E^-(x,\gamma\backslash x))(F(\gamma\backslash x)-F(\gamma)) + \int_{\mathbb{R}^d} E^+(y,\gamma)(F(\gamma\cup y)-F(\gamma))\mathrm{d}y$$

with $m > 0$ and $E^\pm(x,\gamma) = \sum_{y\in\gamma} a^\pm(x-y)$. This model was discussed in [5], where the authors proved local existence of solutions for quasi-observables, and correlation functions. Moreover the existence of evolution of states was shown. In this section we will prove the existence of solutions for quasi-observables in the time dependent case, i.e.

$$\begin{aligned}(L(t)F)(\gamma) &= \sum_{x\in\gamma}(m(t) + E_t^-(x,\gamma\backslash x))(F(\gamma\backslash x) - F(\gamma)) \\ &\quad + \int_{\mathbb{R}^d} E_t^+(y,\gamma)(F(\gamma\cup y) - F(\gamma))\mathrm{d}y\end{aligned}$$

under the following assumptions for $T > 0$

1. m is a continuous non-negative function in $t \in [0,T]$
2. The dispersion and competition kernels $a_t^\pm(x) = a_t^\pm(-x) \ge 0$ are continuous as mappings

$$[0,T] \ni t \longmapsto a_t^\pm \in L^\infty(\mathbb{R}^d), \quad [0,T] \ni t \longmapsto a_t^\pm \in L^1(\mathbb{R}^d).$$

3. There exists a $\Theta > 0$ such that
$$a_t^+(x) \le \Theta a_t^-(x) \tag{28}$$
holds for all $t \in [0,T]$ and almost all $x \in \mathbb{R}^d$.

The last condition (28) means that the dispersion kernel is dominated by the competition kernel uniformly in the time $t \in [0,T]$. The corresponding operator for quasi-observables is formally given by the expressions
$$\hat{L}(t) = A(t) + B(t)$$
with
$$\begin{aligned} A(t) &= A_1(t) + A_2(t) \\ (A_1(t)G)(\eta) &= -E_t(\eta)G(\eta) \\ (A_2(t)G)(\eta) &= \int_{\mathbb{R}^d} E_t^+(y,\eta)G(\eta \cup y)\mathrm{d}y \end{aligned}$$
and
$$\begin{aligned} B(t) &= B_1(t) + B_2(t) \\ (B_1(t)G)(\eta) &= -\sum_{x\in\eta} E_t^-(x,\eta\backslash x)G(\eta\backslash x) \\ (B_2(t)G)(\eta) &= \int_{\mathbb{R}^d} \sum_{x\in\eta} a_t^+(x-y)G(\eta\backslash x \cup y)\mathrm{d}y, \end{aligned}$$
where $E_t(\eta) = \sum\limits_{x\in\eta}(m(t) + E_t^-(x,\eta\backslash x)) = m(t)|\eta| + E_t^-(\eta)$ and $E_t^{\pm}(\eta) = \sum\limits_{x\in\eta} E_t^{\pm}(x,\eta\backslash x)$. As usual we will work in the scale $\mathbb{B}'_\alpha = L^1(\Gamma_0, e^{\alpha|\cdot|}\mathrm{d}\lambda)$, then a simple calculation shows the following result.

Lemma 3.11. *The above expressions define linear bounded operators $A, B \in L(\mathbb{B}'_\alpha, \mathbb{B}'_{\alpha'})$ for $\alpha' < \alpha$ with norm estimates*
$$\|A(t)\|_{\alpha\alpha'} \le \frac{m(t)}{e(\alpha-\alpha')} + \frac{4(\|a_t^-\|_{L^\infty} + \|a_t^+\|_{L^\infty}e^{-\alpha'})}{e^2(\alpha-\alpha')^2} \tag{29}$$
and
$$\|B(t)\|_{\alpha\alpha'} \le \frac{\|a_t^-\|_{L^1}e^{\alpha'} + \|a_t^+\|_{L^1}}{e(\alpha-\alpha')}. \tag{30}$$

In view of Theorem 2.10 we have as a consequence of (30) that $\|B(t)\|_{\alpha\alpha'} \le \dfrac{M}{\alpha-\alpha'}$ for some constant $M = M(\alpha_*, \alpha^*)$ if we fix $\alpha_* < \alpha^*$, cf. Definition 5. Since we cannot apply Theorem 2.10 for the operator A, c.f. (29), the next step for us will be to prove existence of an evolution family corresponding to A in order to apply Theorem 2.13. But first we need to show the continuity of $t \longmapsto A(t)$ and $t \longmapsto B(t)$ in the uniform operator topology. For $\alpha < \alpha'$ consider the mappings
$$\mathbb{R}_+ \times X_+ \times X_+ \longrightarrow L(\mathbb{B}'_\alpha, \mathbb{B}'_{\alpha'}), \quad (m, a^+, a^-) \longmapsto \hat{L}(m, a^+, a^-). \tag{31}$$

with

$$X = \{f \in L^1(\mathbb{R}^d) \cap L^\infty(\mathbb{R}^d) : \quad f(x) = f(-x), \quad \text{for a.a. } x \in \mathbb{R}^d\}.$$

Here X_+ denotes the positive cone of X consisting of all elements $0 \leq f \in X$. The previous Lemma shows, that this map is well-defined. Endow X with the norm

$$\|f\|_X = \max\{\|f\|_{L^1}, \|f\|_{L^\infty}\}$$

so $(X, \|\cdot\|_X)$ is a closed subspace of the Banach space $L^1(\mathbb{R}^d) \cap L^\infty(\mathbb{R}^d)$ and thus a Banach space itself. If we define on the parameter space $\mathbb{R}_+ \times X_+ \times X_+$ the metric

$$d((m, a^+, a^-), (m', b^+, b^-)) = |m - m'| + \|a^+ - b^+\|_X + \|a^- - b^-\|_X$$

the following result holds.

Lemma 3.12. *For $\alpha' < \alpha$ the mapping* (31) *is continuous, where $L(\mathbb{B}'_\alpha, \mathbb{B}'_{\alpha'})$ has the topology induced by the operator norm.*

Proof. Since $\hat{L}$ depend linearly on the coefficients m, a^+, a^- we obtain from Lemma 3.13

$$\begin{aligned} &\|\hat{L}(m, a^+, a^-) - \hat{L}(m', b^+, b^-)\|_{\alpha\alpha'} \\ \leq\ & \frac{4\|a^- - b^-\|_{L^\infty} + 4\|a^+ - b^+\|_{L^\infty} e^{-\alpha'}}{e^2(\alpha - \alpha')^2} \\ &+ \frac{|m - m'| + \|a^- - b^-\|_{L^1} e^{\alpha'} + \|a^+ - b^+\|_{L^1}}{e(\alpha - \alpha')}. \quad \square \end{aligned}$$

The continuity of m, a^+, a^- imply the continuity of

$$[0, T] \ni t \longmapsto (m(t), a_t^+, a_t^-) \in \mathbb{R}_+ \times X_+ \times X_+$$

and as a consequence we obtain the desired continuity of

$$[0, T] \ni t \longmapsto A(t), \quad [0, T] \ni t \longmapsto B(t)$$

in the uniform operator topology on $L(\mathbb{B}'_\alpha, \mathbb{B}'_{\alpha'})$. Now we are prepared to prove the existence of an evolution family corresponding to $A(t)$.

Theorem 3.13. *Let α_* be such that $\Theta e^{-\alpha_*} < 1$ holds. Then for all $\alpha_* \leq \alpha' < \alpha$ there exists a unique evolution family $(\hat{U}(t, s))_{0 \leq s \leq t \leq T}$ on $\mathbb{B}'_{\alpha'}$ satisfying*

1. *$\frac{\partial \hat{U}}{\partial t}(t, s)G = A(t)\hat{U}(t, s)G$ on $\mathbb{B}_{\alpha'}$ for $G \in \mathbb{B}'_\alpha$, in the case of $t = s$ the derivative is meant to be a right-sided derivative.*

2. *$\frac{\partial \hat{U}}{\partial s}(t, s)G = -\hat{U}(t, s)A(s)G$ on $\mathbb{B}_{\alpha'}$ for $G \in \mathbb{B}'_\alpha$.*

Proof. By [5] for each $\alpha_* \leq \alpha'$ there exists a sub stochastic analytic C_0-semigroup $S_t^{\alpha'}(\tau) = e^{\tau A(t)}$ on $\mathbb{B}'_{\alpha'}$. The generator is given by $(A(t), D_{\alpha'}(A(t)))$ with

$$D_{\alpha'}(A(t)) = \{G \in \mathbb{B}'_{\alpha'} : \quad E_t(\cdot)G(\cdot) \in \mathbb{B}'_{\alpha'}\}.$$

For $\alpha_* \leq \alpha' < \alpha$ the part $\widetilde{A}(t)$ of $(A(t), D_{\alpha'}(A(t)))$ on $\mathbb{B}'_\alpha$ is given by

$$\begin{aligned} D(\widetilde{A}(t)) &= \{G \in \mathbb{B}'_\alpha \cap D_{\alpha'}(A(t)) : \quad E_t(\cdot)G(\cdot) \in \mathbb{B}'_\alpha\} \\ &= \{G \in \mathbb{B}'_\alpha : \quad E_t(\cdot)G(\cdot) \in \mathbb{B}'_\alpha\} = D_\alpha(A(t)) \end{aligned}$$

and hence is a generator of a substochastic analytic semigroup, which shows the assumptions of Theorem 2.1. Therefore for $\alpha_* \leq \alpha' < \alpha$ the semigroups satisfy

$$S_t^\alpha(\tau) = S_t^{\alpha'}(\tau)|_{\mathbb{B}'_\alpha}, \quad \forall t \in [0,T] \quad \text{and } \tau \geq 0.$$

Concerning the proof of Theorem 2.1, cf. [24], the evolution families are obtained as limits $\hat{U}^\alpha(t,s) = \lim_{n\to\infty} \hat{U}_n^\alpha(t,s)$ in $\mathbb{B}'_\alpha$. Since $U_n^\alpha(t,s)$ is a composition of $S_t^\alpha(\tau)$

$$\hat{U}_n^\alpha(t,s) = \hat{U}_n^{\alpha'}(t,s)|_{\mathbb{B}'_\alpha}, \quad (t,s) \in \Delta$$

for $\alpha_* \leq \alpha' < \alpha$ follows. To show the property

$$\hat{U}^\alpha(t,s) = \hat{U}^{\alpha'}(t,s)|_{\mathbb{B}'_\alpha} \tag{32}$$

consider for $G \in \mathbb{B}'_\alpha$

$$\begin{aligned} & \|\hat{U}^\alpha(t,s)G - \hat{U}^{\alpha'}(t,s)G\|_{\alpha'} \\ \leq\ & \|\hat{U}^\alpha(t,s)G - \hat{U}_n^\alpha(t,s)G\|_{\alpha'} + \|\hat{U}_n^\alpha(t,s)G - \hat{U}^{\alpha'}(t,s)G\|_{\alpha'} \\ \leq\ & \|\hat{U}^\alpha(t,s)G - \hat{U}_n^\alpha(t,s)G\|_{\alpha} + \|\hat{U}_n^{\alpha'}(t,s)G - \hat{U}^{\alpha'}(t,s)G\|_{\alpha'} \end{aligned}$$

and take $n \to \infty$. Hence $\hat{U}^\alpha(t,s)G = \hat{U}^{\alpha'}(t,s)G$ in $\mathbb{B}'_{\alpha'}$ and therefore by definition of the norm also pointwise for a.a. $\eta \in \Gamma_0$, which implies (32) in $\mathbb{B}'_\alpha$. Now (32) implies the conditions for Theorem 2.3 and hence the desired result. □

Corollary 3.14. *Let α_* be such that $\Theta e^{-\alpha_*} < 1$ and fix some $\alpha^* > \alpha_*$. Then there exists a continuous function $T(\alpha)$ monotonically decreasing and for each $G_0 \in \mathbb{B}'_{\alpha^*}$ a unique solution G_t of*

$$\frac{\mathrm{d}G_t}{\mathrm{d}t} = \hat{L}(t)G_t, \quad G_t|_{t=0} = G_0$$

in the scale $\mathbb{B}'_\alpha$ given by Remark 2.15.3.

3.4 Glauber-type Dynamics in Continuum

The non-equilibrium Glauber-type dynamics can be described by the heuristic Markov pre-generator

$$(LF)(\gamma) = m\sum_{x\in\gamma}(F(\gamma\backslash x) - F(\gamma)) + z\int_{\mathbb{R}^d}(F(\gamma\cup x) - F(\gamma))\exp(-E(x,\gamma))\mathrm{d}x.$$

Let $\phi : \mathbb{R}^d \longrightarrow \mathbb{R}_+$ be an even non-negative function. For any $\gamma \in \Gamma$, $x \in \mathbb{R}^d\backslash\gamma$ we set $E(x,\gamma) = \sum_{y\in\gamma}\phi(x-y) \in [0,\infty]$. Here $z > 0$ is an activity parameter and $m > 0$ is a mortality rate. As before each particle may die according to the rate m. New particles are influenced by existing particles, which is described by the potential ϕ. Big values of ϕ lead to a small factor $e^{-E(x,\gamma)}$ and hence to smaller probabilities for new particles to appear in the regions where $E(x,\gamma)$ is big. The operator for quasi-observables is given by

$$(\hat{L}G)(\eta) = -|\eta| m G(\eta) + z \sum_{\xi \subset \eta} \int_{\mathbb{R}^d} e^{-E(x,\xi)} G(\xi \cup x) e_\lambda(e^{-\phi(x-\cdot)} - 1, \eta \backslash \xi) \mathrm{d}x.$$

The existence of a C_0-semigroup associated to $\hat{L}$ was shown in [17]. In [6], it was proven that this semigroup can be approximated uniformly on compact time intervals using discretization of time. Solutions in scales of Banach spaces were studied in [4] and [11]. This part will partially generalize the results to time dependent coefficients. Likewise the evolution of correlation functions and states will be studied. The evolution equation for correlation functions is given by the operator

$$(L^\Delta k)(\eta) = -|\eta| m k(\eta) + z \sum_{x \in \eta} e^{-E(x, \eta \backslash x)} \int_{\Gamma_0} e_\lambda(t_x, \xi) k((\eta \backslash x) \cup \xi) \mathrm{d}\lambda(\xi) \quad (33)$$

with $t_x(y) = e^{-\phi(x-y)} - 1$. In [6] the existence of correlation function evolution was proven by discretization and further ergodicity properties were studied. We will be concerned with the time dependent case $z = z(t), m = m(t)$ and $\phi = \phi_t$. Starting again with the equation for quasi-observables in the scale $\mathbb{B}'_\alpha = L^1(\Gamma_0, e^{\alpha|\cdot|})$ of type 2 we will impose the following conditions to hold for some $T > 0$

1. $[0,T] \ni t \longmapsto z(t) \geq 0$, $[0,T] \ni t \longmapsto m(t) \geq 0$ are continuous;
2. $\phi_t(x) = \phi_t(-x) \geq 0$ is a continuous mapping in the sense that

$$[0,T] \ni t \longmapsto \phi_t \in L^\infty(\mathbb{R}^d), \quad [0,T] \ni t \longmapsto \phi_t \in L^1(\mathbb{R}^d)$$

   is continuous;
3. there exists a potential $\phi(x) = \phi(-x) \geq 0$ such that $\phi_t(x) \leq \phi(x)$ and

$$\beta = \int_{\mathbb{R}^d} (1 - e^{-\phi(x)}) \mathrm{d}x < \infty.$$

Note that 3. implies $1 - e^{-\phi_t(x-\cdot)} \leq 1 - e^{-\phi(x-\cdot)}$ and hence $\int_{\mathbb{R}^d} 1 - e^{-\phi_t(x)} \mathrm{d}x = \beta_t \leq \beta < \infty$. The last condition is important to have uniform bounds in the time variable t. As a first step we will show continuity properties of

$$\begin{aligned}(\hat{L}(t)G)(\eta) &= -|\eta| m(t) G(\eta) \\ &\quad + z(t) \sum_{\xi \subset \eta} \int_{\mathbb{R}^d} e^{-E_t(x,\xi)} G(\xi \cup x) e_\lambda(e^{-\phi_t(x-\cdot)} - 1, \eta \backslash \xi) \mathrm{d}x.\end{aligned} \quad (34)$$

Lemma 3.15. *Under conditions 1-3. expression* (34) *defines a bounded linear operator* $\hat{L}(t) \in L(\mathbb{B}'_\alpha, \mathbb{B}'_{\alpha'})$ *for* $\alpha' < \alpha$ *satisfying*

$$\|\hat{L}(t)\|_{\alpha\alpha'} \leq \frac{m(t) + z(t)\exp\left(e^{\alpha'}\beta_t\right)e^{-\alpha'}}{e(\alpha-\alpha')}.$$

Further the mapping $[0,T] \ni t \longmapsto \hat{L}(t) \in L(\mathbb{B}'_\alpha, \mathbb{B}'_{\alpha'})$ *is continuous in the uniform operator topology.*

Proof. For $\alpha' < \alpha$ and $G \in \mathbb{B}'_\alpha$ we obtain

$$\begin{aligned}
&\int_{\Gamma_0} \sum_{\xi\subset\eta} \int_{\mathbb{R}^d} e^{-E_t(x,\xi)} e_\lambda(|t_x|, \eta\backslash\xi)|G(\xi\cup x)|e^{\alpha'|\eta|}\mathrm{d}x\mathrm{d}\lambda(\eta) \\
= &\int_{\Gamma_0}\int_{\Gamma_0}\int_{\mathbb{R}^d} e^{-E_t(x,\xi)} e_\lambda(|t_x|;\eta)|G(\xi\cup x)|e^{\alpha'|\eta|}e^{\alpha'|\xi|}\mathrm{d}x\mathrm{d}\lambda(\xi)\mathrm{d}\lambda(\eta) \\
\leq &\exp\left(e^{\alpha'}\beta_t\right)\int_{\Gamma_0}\int_{\mathbb{R}^d} |G(\xi\cup x)|e^{\alpha'|\xi|}\mathrm{d}x\mathrm{d}\lambda(\xi) \\
\leq &\frac{\exp\left(e^{\alpha'}\beta_t\right)e^{-\alpha'}}{e(\alpha-\alpha')}\|G\|_\alpha
\end{aligned}$$

which shows the first assertion. For the second part of the assertion of the lemma take $G \in \mathbb{B}'_\alpha$ and $t, s \in [0,T]$, then we have for the death part

$$|m(t) - m(s)| \int_{\Gamma_0} |\eta||G(\eta)|e^{\alpha'|\eta|}\mathrm{d}\lambda(\eta) \leq \frac{|m(t)-m(s)|}{e(\alpha-\alpha')}\|G\|_\alpha,$$

which has the desired property. The birth part can be estimated by (+)

$$\begin{aligned}
&z(t)\int_{\Gamma_0}\int_{\Gamma_0}\int_{\mathbb{R}^d}\left|e_\lambda(1-e^{-\phi_t(x-\cdot)},\eta) - e_\lambda(1-e^{-\phi_s(x-\cdot)},\eta)\right| \\
&\qquad\times|G(\xi\cup x)|e^{\alpha'|\eta\cup\xi|}\mathrm{d}x\mathrm{d}\lambda(\xi)\mathrm{d}\lambda(\eta) \\
+\;&z(t)\int_{\Gamma_0}\int_{\Gamma_0}\int_{\mathbb{R}^d} e_\lambda(1-e^{-\phi_s(x-\cdot)},\eta)\left|e^{-E_t(x,\xi)} - e^{-E_s(x,\xi)}\right| \\
&\qquad\times|G(\xi\cup x)|e^{\alpha'|\eta\cup\xi|}\mathrm{d}x\mathrm{d}\lambda(\xi)\mathrm{d}\lambda(\eta) \\
+\;&|z(t)-z(s)|\int_{\Gamma_0}\int_{\Gamma_0}\int_{\mathbb{R}^d} e_\lambda(1-e^{-\phi_s(x-\cdot)},\eta)|G(\xi\cup x)|e^{\alpha'|\eta\cup\xi|}\mathrm{d}x\mathrm{d}\lambda(\xi)\mathrm{d}\lambda(\eta).
\end{aligned}$$

Using

$$\begin{aligned}
&\left|e_\lambda(e^{-\phi_t(x-\cdot)}-1,\eta) - e_\lambda(e^{-\phi_s(x-\cdot)}-1,\eta)\right| \\
\leq &\sum_{y\in\eta}|e^{-\phi_t(x-y)} - e^{-\phi_s(x-y)}|e_\lambda(1-e^{-\phi(x-\cdot)},\eta\backslash y) \\
\leq &\sum_{y\in\eta}|\phi_t(x-y) - \phi_s(x-y)|e_\lambda(1-e^{-\phi(x-\cdot)},\eta\backslash y)
\end{aligned}$$

we estimate the first part of (+) by

$$
\begin{aligned}
& z(t) \int_{\Gamma_0} \int_{\mathbb{R}^d} \int_{\Gamma_0} \sum_{y \in \eta} |\phi_t(x-y) - \phi_s(x-y)| e_\lambda(1 - e^{-\phi(x-\cdot)}, \eta \backslash y) \\
& \qquad \times |G(\xi \cup x)| e^{\alpha' |\eta \cup \xi|} \mathrm{d}\lambda(\eta) \mathrm{d}x \mathrm{d}\lambda(\xi) \\
= \; & z(t) e^{\alpha'} \|\phi_t - \phi_s\|_{L^1} \int_{\Gamma_0} \int_{\mathbb{R}^d} \int_{\Gamma_0} e_\lambda(1 - e^{-\phi(x-\cdot)}, \eta) e^{\alpha' |\eta|} \mathrm{d}\lambda(\eta) \\
& \qquad \times |G(\xi \cup x)| e^{\alpha' |\xi|} \mathrm{d}x \mathrm{d}\lambda(\xi) \\
= \; & z(t) e^{\alpha'} \|\phi_t - \phi_s\|_{L^1} \exp(e^{\alpha'} \beta) \int_{\Gamma_0} \int_{\mathbb{R}^d} |G(\xi \cup x)| e^{\alpha' |\xi|} \mathrm{d}x \mathrm{d}\lambda(\xi) \\
\leq \; & \frac{z(t) \|\phi_t - \phi_s\|_{L^1} \exp(e^{\alpha'} \beta)}{e(\alpha - \alpha')} \|G\|_\alpha.
\end{aligned}
$$

Because of

$$
\left| e^{-E_t(x,\xi)} - e^{-E_s(x,\xi)} \right| \leq |E_s(x,\xi) - E_t(x,\xi)| \leq |\xi| \|\phi_t - \phi_s\|_{L^\infty}
$$

we obtain for the second part of (+)

$$
\begin{aligned}
& z(t) \|\phi_t - \phi_s\|_\infty \int_{\Gamma_0} \int_{\Gamma_0} \int_{\mathbb{R}^d} e_\lambda(1 - e^{-\phi_s(x-\cdot)}, \eta) e^{\alpha' |\eta|} \\
& \qquad \times |G(\xi \cup x)| |\xi| e^{\alpha' |\xi|} \mathrm{d}x \mathrm{d}\lambda(\xi) \mathrm{d}\lambda(\eta) \\
\leq \; & z(t) \|\phi_t - \phi_s\|_\infty \exp(e^{\alpha'} \beta_s) e^{-\alpha'} \int_{\Gamma_0} |\xi|^2 |G(\xi)| e^{\alpha' |\xi|} \mathrm{d}\lambda(\xi) \\
\leq \; & \frac{4 z(t) \|\phi_t - \phi_s\|_\infty e^{-\alpha'}}{e^2 (\alpha - \alpha')^2} \exp(e^{\alpha'} \beta_s) \|G\|_\alpha.
\end{aligned}
$$

For the last part of (+) we get

$$
\begin{aligned}
& |z(t) - z(s)| \int_{\Gamma_0} \int_{\Gamma_0} \int_{\mathbb{R}^d} e_\lambda(1 - e^{-\phi_s(x-\cdot)}, \eta) |G(\xi \cup x)| e^{\alpha' |\xi|} e^{\alpha' |\eta|} \mathrm{d}x \mathrm{d}\lambda(\xi) \mathrm{d}\lambda(\eta) \\
\leq \; & \frac{|z(t) - z(s)| e^{-\alpha'}}{e(\alpha - \alpha')} \exp(e^{\alpha'} \beta_s) \|G\|_\alpha
\end{aligned}
$$

which proves the assertion. □

Since $\|\hat{L}(t)\|_{\alpha'\alpha} \leq \dfrac{M}{\alpha - \alpha'}$, with

$$
M = \frac{\overline{m} + \overline{z} \exp\left(e^{\alpha'} \beta \right) e^{-\alpha'}}{e}
$$

$\overline{m} = \sup_{t \geq 0} m(t)$ and $\overline{z} = \sup_{t \geq 0} z(t)$ we can apply Theorem 2.10 and prove the existence of solutions in the scale $\mathbb{B}'_\alpha$. For the time independent parameters the existence was proved directly in [4].

Theorem 3.16. *Under conditions 1-3. and for fixed $\alpha_* < \alpha^*$ there exists $T : [\alpha_*, \alpha^*) \longrightarrow [0, T]$ continuous and monotonically decreasing, such that for each $G_0 \in \mathbb{B}'_{\alpha^*} = L^1(\Gamma_0, e^{\alpha|\cdot|}d\lambda)$ there exists a unique solution G_t to the Cauchy problem*

$$\frac{\partial G_t}{\partial t} = \hat{L}(t)G_t, \quad G_t|_{t=0} = G_0 \tag{35}$$

in the scale $\mathbb{B}'_\alpha$ given by Remark 2.12.3

Proof. Lemma 3.18 implies

$$\begin{aligned}\|\hat{L}(t)\|_{\alpha\alpha'} &\leq \frac{m(t) + z(t)\exp\left(e^{\alpha'}\beta_t\right)e^{-\alpha'}}{e(\alpha - \alpha')} \\ &\leq \frac{\overline{m} + \overline{z}\exp\left(e^{\alpha^*}\beta\right)e^{-\alpha_*}}{e(\alpha - \alpha')}\end{aligned}$$

with $\overline{m} = \sup\limits_{t\in[0,T]} m(t)$ and $\overline{z} = \sup\limits_{t\in[0,T]} z(t)$, which shows the first assumption of Theorem 2.10. Since continuity in the uniform operator topology implies strong continuity Theorem 2.10 is applicable and shows the existence of unique solutions to (35). □

Likewise using the same techniques we can prove existence of solutions for the corresponding equations for correlation functions, c.f. (33). First we show general properties of operators $L^\Delta(t)$ in the scale of Banach spaces $\mathbb{B}_\alpha = L^\infty(\Gamma_0, e^{-\alpha|\cdot|}d\lambda)$.

Lemma 3.17. *Under conditions 1-3. the expression*

$$\begin{aligned}(L^\Delta(t)k)(\eta) &= -|\eta|m(t)k(\eta) \\ &\quad + z(t)\sum_{x\in\eta} e^{-E_t(x,\eta\backslash x)} \int_{\Gamma_0} e_\lambda(e^{-\phi_t(x-\cdot)} - 1, \xi)k((\eta\backslash x)\cup\xi)d\lambda(\xi)\end{aligned}$$

defines an operator $L^\Delta(t) \in L(\mathbb{B}_{\alpha'}, \mathbb{B}_\alpha)$ for $\alpha' < \alpha$ such that

$$\|L^\Delta(t)\|_{\alpha'\alpha} \leq \frac{m(t) + z(t)e^{-\alpha'}\exp(e^{\alpha'}\beta_t)}{e(\alpha - \alpha')}.$$

Moreover, the mapping $[0, T] \ni t \longmapsto L^\Delta(t) \in L(\mathbb{B}_{\alpha'}, \mathbb{B}_\alpha)$ is continuous in the uniform operator topology.

Proof. Let $\alpha' < \alpha$ and $k \in \mathbb{B}_{\alpha'}$ be fixed, then the first summand gives

$$|\eta|m(t)|k(\eta)|e^{-\alpha|\eta|} \leq m(t)\|k\|_{\alpha'}|\eta|e^{-(\alpha-\alpha')|\eta|} \leq \frac{m(t)}{e(\alpha - \alpha')}\|k\|_{\alpha'}$$

and for the second part

$$\begin{aligned} & z(t)\sum_{x\in\eta} e^{-E_t(x,\eta\backslash x)} \int_{\Gamma_0} e_\lambda(1-e^{-\phi_t(x-\cdot)},\xi)|k(\eta\backslash x\cup\xi)|\mathrm{d}\lambda(\xi)e^{-\alpha|\eta|} \\ \leq\ & z(t)\|k\|_{\alpha'}e^{-\alpha'}e^{-(\alpha-\alpha')|\eta|}\sum_{x\in\eta}\int_{\Gamma_0} e_\lambda(1-e^{-\phi_t(x-\cdot)},\xi)e^{\alpha'|\xi|}\mathrm{d}\lambda(\xi) \\ =\ & z(t)\|k\|_{\alpha'}e^{-\alpha'}|\eta|e^{-(\alpha-\alpha')|\eta|}\exp(e^{\alpha'}\beta_t) \leq \frac{z(t)e^{-\alpha'}\exp(e^{\alpha'}\beta_t)}{e(\alpha-\alpha')}\|k\|_{\alpha'}. \end{aligned}$$

Thus the first claim is proved. For the second part let $t,s\in[0,T]$ be arbitrary, then the death part can be estimated by

$$|\eta||m(t)-m(s)|e^{-\alpha|\eta|}|k(\eta)| \leq |m(t)-m(s)||\eta|e^{-(\alpha-\alpha')|\eta|}\|k\|_{\alpha'} \leq \frac{|m(t)-m(s)|}{e(\alpha-\alpha')}\|k\|_{\alpha'}.$$

Analogously to Lemma 3.18 the birth part can be estimated by

$$\begin{aligned} & |z(t)-z(s)|\sum_{x\in\eta} e^{-E_t(x,\eta\backslash x)} \int_{\Gamma_0} e_\lambda(1-e^{-\phi_t(x-\cdot)},\xi)|k(\eta\backslash x\cup\xi)|\mathrm{d}\lambda(\xi)e^{-\alpha|\eta|} \\ & +z(s)\sum_{x\in\eta}\left|e^{-E_t(x,\eta\backslash x)}-e^{-E_s(x,\eta\backslash x)}\right| \\ & \quad\times\int_{\Gamma_0} e_\lambda(1-e^{-\phi_t(x-\cdot)},\xi)|k(\eta\backslash x\cup\xi)|\mathrm{d}\lambda(\xi)e^{-\alpha|\eta|} \\ & +z(s)\sum_{x\in\eta} e^{-E_s(x,\eta\backslash x)} \\ & \quad\times\int_{\Gamma_0}\left|e_\lambda(1-e^{-\phi_t(x-\cdot)},\xi)-e_\lambda(1-e^{-\phi_s(x-\cdot)},\xi)\right||k(\eta\backslash x\cup\xi)|\mathrm{d}\lambda(\xi)e^{-\alpha|\eta|}. \end{aligned}$$

The first summand can be bounded by

$$\begin{aligned} & |z(t)-z(s)|e^{-\alpha'}|\eta|e^{-(\alpha-\alpha')|\eta|}\exp(e^{\alpha'}\beta_t)\|k\|_{\alpha'} \\ \leq\ & \frac{|z(t)-z(s)|e^{-\alpha'}\exp(e^{\alpha'}\beta)}{e(\alpha-\alpha')}\|k\|_{\alpha'} \end{aligned}$$

and the second one by

$$\begin{aligned} & z(s)\|\phi_t-\phi_s\|_\infty e^{-\alpha'}|\eta|^2e^{-(\alpha-\alpha')|\eta|}\exp(e^{\alpha'}\beta_t)\|k\|_{\alpha'} \\ \leq\ & \frac{4z(s)\|\phi_t-\phi_s\|_\infty\exp(e^{\alpha'}\beta_t)}{e^2(\alpha-\alpha')^2}\|k\|_{\alpha'}. \end{aligned}$$

As a result they have desired property. For the last term we have the following estimate

$$z(s)\sum_{x\in\eta}\int_{\Gamma_0}\sum_{y\in\xi}|\phi_t(x-y)-\phi_s(x-y)|e_\lambda(1-e^{-\phi(x-\cdot)},\xi\backslash y)|k(\eta\backslash x\cup\xi)|e^{-\alpha|\eta|}\mathrm{d}\lambda(\xi)$$

$$\leq z(s)\|k\|_{\alpha'}e^{-(\alpha-\alpha')|\eta|}\|\phi_t-\phi_s\|_{L^1}\sum_{x\in\eta}\int_{\Gamma_0}e_\lambda(1-e^{-\phi(x-\cdot)},\xi)e^{\alpha'|\xi|}\mathrm{d}\lambda(\xi)$$

$$\leq\frac{z(s)\|\phi_t-\phi_s\|_{L^1}\exp(e^{\alpha'}\beta)}{e(\alpha-\alpha')}\|k\|_{\alpha'}$$

which shows the continuity. □

As a consequence, by Theorem 2.10 we obtain the existence of local solutions.

Theorem 3.18. *Fix some $\alpha_*<\alpha^*$, then there exists $T:(\alpha^*,\alpha^*]\longrightarrow[0,T]$ continuous and monotonically increasing such that for each $k_0\in\mathbb{B}_{\alpha_*}$ there exists a unique solution k_t to the Cauchy problem*

$$\frac{\partial k_t}{\partial t}(t)=L^\Delta(t)k_t,\quad k_t|_{t=0}=k_0 \tag{36}$$

in the scale $\mathbb{B}_\alpha$ given by Remark 2.12.3.

To have the existence of a solution via evolution families it is sufficient to show that the operators $L^\Delta(t)$ generate contraction semigroups $T_t^\Delta(s)$ for $t\in[0,T]$. Since the scale $\mathbb{B}_\alpha$ is of L^∞-type it is not straightforward. The general approach is to consider the dual semigroups and show the existence of appropriate invariant subspaces. This analysis could be done, but is not the purpose of this work. Instead we will consider the evolution of Bogoliubov generating functionals. The fact that

$$\sup_{x\in\mathbb{R}^d}\left|\frac{e^{hx}-1}{h}-x\right|=\infty,\quad\forall h>0$$

causes difficulties in many calculations. Therefore we will only consider the simplified model with the time independent potential ϕ. Let m and z be continuous functions on some interval $I=[0,T]$ and $\phi(x)=\phi(-x)\geq 0$ be integrable, i.e.,

$$\beta=\int_{\mathbb{R}^d}1-e^{-\phi(x)}\mathrm{d}x\leq\int_{\mathbb{R}^d}\phi(x)\mathrm{d}x=\|\phi\|_{L^1}.$$

In [11] it was shown, that the generator $\widetilde{L}(t)$ for fixed $t\in[0,T]$ is given by

$$\begin{aligned}(\widetilde{L}(t)B)(\Theta) &= -\int_{\mathbb{R}^d}\Theta(x)\Big(m(t)\delta B(\Theta;x)-z(t)B(\Theta e^{-\phi(x-\cdot)}+e^{-\phi(x-\cdot)}-1)\Big)\mathrm{d}x\\ &= m(t)(L_0B)(\Theta)+z(t)(L_1B)(\Theta)\end{aligned}$$

with

$$(L_0B)(\Theta)=-\int_{\mathbb{R}^d}\Theta(x)\delta B(\Theta;x)\mathrm{d}x$$

and

$$(L_1 B)(\Theta) = \int_{\mathbb{R}^d} \Theta(x) B\left(\Theta e^{-\phi(x-\cdot)} + e^{-\phi(x-\cdot)} - 1)\right) \mathrm{d}x.$$

It is a simple matter to show that

$$\|\widetilde{L}(t)\|_{\alpha\alpha'} \leq \frac{\alpha^* \left(m(t) + z(t)\alpha^* \exp\left(\frac{\|\phi\|_{L^1}}{\alpha_*} - 1\right)\right)}{\alpha - \alpha'} \tag{37}$$

where the norm of the operator is taken in $L(\mathbb{B}'_\alpha, \mathbb{B}'_{\alpha'})$ with $\alpha_* \leq \alpha' < \alpha \leq \alpha^*$ and $\mathbb{B}'_\alpha$ is defined in (17). This bound was shown in [11] for the case $m \equiv 1$.

Theorem 3.19. *Let m, z be continuous on $[0, T]$ and $0 \leq \phi \in L^1(\mathbb{R}^d)$ be symmetric. Then for each fixed $0 < \alpha_* < \alpha^*$ there exists a continuous and monotonically decreasing function $T : [\alpha_*, \alpha^*) \longrightarrow [0, T]$ such that for each $B_0 \in \mathbb{B}'_{\alpha^*}$ there exists a unique solution B_t of the Cauchy problem*

$$\frac{\partial B_t}{\partial t} = \widetilde{L}(t) B_t, \quad B_t|_{t=0} = B_0$$

in the scale $\mathbb{B}'_\alpha$ given by Remark 2.12.3.

Proof. Previous results, cf. (37), show that $\|\tilde{L}(t)\|_{\alpha\alpha'} \leq \frac{M}{\alpha - \alpha'}$ for some constant $M > 0$ independent of $t \in [0, T]$. Strong continuity follows from the inequality

$$\|\widetilde{L}(t)B - \widetilde{L}(s)B\|_{\alpha'} \leq |m(t) - m(s)| \|L_0 B\|_{\alpha'} + |z(t) - z(s)| \|L_1 B\|_{\alpha'}$$

for $\alpha' < \alpha$, $t, s \in [0, T]$, $B \in \mathbb{B}'_\alpha$ and the fact $L_0, L_1 \in L(\mathbb{B}'_\alpha, \mathbb{B}'_{\alpha'})$, which was shown in [11]. An application of Theorem 2.10 shows the existence of a unique evolution B_t in the scale $\mathbb{B}'_\alpha$. □

3.5 General birth-and-death dynamics

The aim of the last section is to prove the existence of solutions for the evolution of quasi-observables for the general birth-and-death dynamics heuristically given by the Markov pre-generator

$$\begin{aligned}(L(t)F)(\gamma) &= m(t) \sum_{x \in \gamma} d(x, \gamma \backslash x)(F(\gamma \backslash x) - F(\gamma)) \\ &\quad + \kappa(t) \int_{\mathbb{R}^d} b(x, \gamma)(F(\gamma \cup x) - F(\gamma)) \mathrm{d}x.\end{aligned}$$

For time independent m and κ this model was discussed recently in [8]. Under some conditions the authors proved the existence of evolution for quasi-observables via semigroup techniques. We will use this result together with Theorem 2.3 to construct an evolution of quasi-observables for time dependent coefficients $m = m(t)$ and $\kappa = \kappa(t)$. The assumptions on the model are the following

1. m, κ are non-negative, continuous on $\mathbb{R}_+$ and bounded.
2. $d(x,\gamma) \geq 0$ and $b(x,\gamma) \geq 0$ are locally integrable in $\eta \in \Gamma_0$, i.e.,
$$\int\limits_{\Gamma_{0,\Lambda}^{(n)}} d(x,\eta) + b(x,\eta)\mathrm{d}\lambda(\eta) < \infty$$
for all $n \in \mathbb{N}$, $\Lambda \in \mathscr{B}_c(\mathbb{R}^d)$ and $x \in \mathbb{R}^d$
3. There exists $\alpha^* \in \mathbb{R}$ and $a_1 \geq 1$ such that for all $\xi \in \Gamma_0$ and $x \in \mathbb{R}^d$
$$\sum_{x\in\xi}\int\limits_{\Gamma_0} |K^{-1}d(x,\cdot\cup\xi\backslash x)|(\eta)e^{\alpha^*|\eta|}\mathrm{d}\lambda(\eta) \leq a_1 D(\xi)$$
4. There exists $a_2 > 0$ such that for all $\xi \in \Gamma_0$ and $x \in \mathbb{R}^d$
$$\sum_{x\in\xi}\int\limits_{\Gamma_0} |K^{-1}b(x,\cdot\cup\xi\backslash x)|(\eta)e^{\alpha^*|\eta|}\mathrm{d}\lambda(\eta) \leq a_2 D(\xi)$$
5. There exists a constant $\nu > 0$ and $A > 0$ for which
$$d(x,\eta\backslash x) \leq Ae^{\nu|\eta|}$$
holds for each $\eta \in \Gamma_0$ and $x \in \mathbb{R}^d$.

The bound on d implies the bound

$$D(\eta) = \sum_{x\in\eta} d(x,\eta\backslash x) \leq A|\eta|e^{\nu|\eta|} \tag{38}$$

on D. Of course 5. can be replaced by $d(x,\eta\backslash x) \leq P(|\eta|)e^{\nu|\eta|}$ with P a polynomial. The expressions for quasi-observables are given by

$$\begin{aligned}
& (\hat{L}(t)G)(\eta) \\
= \; & -m(t)\sum_{\xi\subset\eta} G(\xi)\sum_{x\in\xi}(K^{-1}d(x,\cdot\cup\xi\backslash x))(\eta\backslash\xi) \\
& +\kappa(t)\sum_{\xi\subset\eta}\int\limits_{\mathbb{R}^d} G(\xi\cup x)(K^{-1}b(x,\cdot\cup\xi))(\eta\backslash\xi)\mathrm{d}x \\
= \; & m(t)\hat{L}_0 + \hat{L}_1(t)
\end{aligned}$$

with $\hat{L}_1(t) = \hat{L}(t) - m(t)\hat{L}_0$ and $(L_0G)(\eta) = -D(\eta)G(\eta)$, where

$$D(\eta) = \sum_{x\in\eta} d(x,\eta\backslash x).$$

As a first step we will show that $\hat{L}(t)$ can be realized as bounded linear operators on $L(\mathbb{B}'_\alpha, \mathbb{B}'_{\alpha'})$.

Lemma 3.20. *$\hat{L}$ defines a bounded linear operator $\hat{L}(t) \in L(\mathbb{B}'_\alpha, \mathbb{B}'_{\alpha'})$ for $\alpha' + \nu < \alpha \le \alpha^*$ with*

$$\|\hat{L}(t)\|_{\alpha\alpha'} \le A\frac{m(t)a_1 + \kappa(t)a_2 e^{-\alpha'}}{\alpha - \alpha' - \nu}. \tag{39}$$

Moreover $\mathbb{R}_+ \ni t \longmapsto \hat{L}(t)$ is continuous in the uniform operator topology.

Proof. In [8] the authors have shown that $\hat{L}_1$ is relatively bounded with respect to $\hat{L}_0$. Similar calculations show that

$$\|\hat{L}_1(t)G\|_{\alpha'} \le (m(t)a_1 + \kappa(t)a_2 e^{-\alpha'} - m(t))\|\hat{L}_0 G\|_{\alpha'}.$$

Using (38) we obtain for $G \in \mathbb{B}'_\alpha$ with $\alpha' < \alpha$

$$\begin{aligned} \|\hat{L}_0 G\|_{\alpha'} &\le \int_{\Gamma_0} D(\eta)|G(\eta)|e^{\alpha'|\eta|}\mathrm{d}\lambda(\eta) \\ &\le A\int_{\Gamma_0} |G(\eta)|e^{\alpha|\eta|}|\eta|e^{-(\alpha-\alpha'-\nu)|\eta|}\mathrm{d}\lambda(\eta) \\ &\le \frac{A}{\alpha - \alpha' - \nu}\|G\|_\alpha \end{aligned}$$

for $\alpha > \alpha' + \nu$. Therefore

$$\|\hat{L}(t)\|_{\alpha\alpha'} \le m(t)\|\hat{L}_0\|_{\alpha\alpha'} + \|\hat{L}_1(t)\|_{\alpha\alpha'} \le A\frac{m(t)a_1 + \kappa(t)a_2 e^{-\alpha'}}{\alpha - \alpha' - \nu}$$

shows $\hat{L}(t) \in L(\mathbb{B}'_\alpha, \mathbb{B}'_{\alpha'})$. Continuity follows from the continuity of m, κ and the linear dependence on the parameters. □

(38) shows that it is possible to realise $\hat{L}_0$ and $\hat{L}(t)$ as an operator with the domain

$$\mathrm{Dom}(\hat{L})_\alpha = \{G \in \mathbb{B}'_\alpha : \quad D(\cdot)G(\cdot) \in \mathbb{B}'_\alpha\}$$

for $\alpha \le \alpha^*$.

Theorem 3.21. *Assume there exists $\alpha_* < \alpha^*$ satisfying*

$$a_1\overline{m} + a_2\overline{\kappa}e^{-\alpha_*} < \frac{3}{2},$$

where $\overline{m} = \sup\limits_{t\ge0} m(t)$ and $\overline{\kappa} = \sup\limits_{t\ge0} \kappa(t)$. Then there exists for each $\alpha_* \le \alpha' < \alpha \le \alpha^*$; $\alpha - \alpha' > \nu$ a unique evolution family $((\hat{U}(t,s))_{0\le s\le t}$ on $\mathbb{B}'_{\alpha'}$. Consequently for each $G_s \in \mathbb{B}'_\alpha$ the equation

$$\frac{\partial G_t}{\partial t} = \hat{L}(t)G_t, \quad s \le t, \quad G_t|_{t=s} = G_s$$

has a unique $\mathbb{B}'_\alpha-$valued solution $G_t = \hat{U}(t,s)G_s$ on $\mathbb{B}'_{\alpha'}$.

Proof. Last lemma implies that by [8] for each $\alpha_* \le \alpha \le \alpha^*$ there exists a unique holomorphic C_0-semigroup $(\hat{S}_t^\alpha(s))_{s\ge 0}$ with the generator $(\hat{L}(t), \mathrm{Dom}_\alpha(\hat{L}))$. The same arguments as in the proof of Theorem 3.15 show $\mathbb{B}'_{\alpha''}$-admissiblility for $\alpha < \alpha''$. The proof in [8] shows that this semigroup is a contraction semigroup on $\mathbb{R}_+$ which implies Kato-stability. Theorem 2.1 implies the existence of a unique evolution family and using again the same arguments as in the proof of Theorem 3.15 one can show that Theorem 2.3 is applicable. □

Remark 3.3. The reason to consider this simple case for the time dependent birth and death coefficients is the continuity of $t \longmapsto \hat{L}(t)$. For more general coefficients d_t and b_t one needs different assumptions, especially for the continuity.

3.6 Conclusion

Concerning correlation functions the major part is to construct an evolution family corresponding to the operator $A(t)$, which does not satisfy the bound $\|A(t)\|_{\alpha'\alpha} \le \dfrac{M}{\alpha - \alpha'}$. Since the embeddings $\mathbb{B}_{\alpha'} \subset \mathbb{B}_\alpha$ are not dense for $\alpha' < \alpha$ it is not possible to apply Theorem 2.1 or Theorem 2.3. To overcome this problem in the time independent case it is possible to show via perturbation techniques, that A generates a C_0-semigroup $S(t)$, cf. [6] and [5], and afterwards consider the dual semigroup $S^*(t)$. Since the Banach spaces we are dealing with are not reflexive, the semigroup $S^*(t)$ will be in general only weak*-continuous. As shown in [21], one can restrict $S^*(t)$ to some invariant subspace $D(S^\odot)$ and obtain again a C_0-semigroup, the so-called sun-dual $S^\odot(t)$. To tackle the problem in the time dependent case we would propose to realize a similar approach for evolution families $U(t,s)$. One difficulty is that $A(t)U(t,s) = U(t,s)A(t)$ does not hold in general. The major question is how to characterize some invariant subspace $D(U^\odot)$ such that $D(U^\odot) \subset \bigcap_{t\in I} D(L(t))$ holds.

To show existence of global solutions we use general results for evolution families. Since they are not applicable for correlation functions further analysis is required. Special properties of the Banach spaces $\mathbb{B}_\alpha$ and of the operators $\hat{L}(t)$ and $L^\Delta(t)$ might be useful to prove approximation formulas in the spirit of [13, 22] and [23]. Consequently, such formulas might allow us to show the existence of an evolution of states. We should stress that only sub-poissonian solutions were considered, but in many applications clustering may appear and therefore the time evolution should also be considered in other classes of functions. Further steps can be dealing with Vlasov-scaling and existence of solutions for the corresponding equations. A next step of generalization is to deal with randomness in this models, meaning that the coefficients $z, m, a^\pm, d$ and b should be random variables. One motivation is the fact that in applications it is not possible to precisely measure the corresponding rates, but also fluctuations could be taken into account. For applications it is important to understand the properties of the solutions of our equations. Like in [**?**] one could analyze properties of solutions to integro-differential equations of the form (26) or even non-linear versions. The case of periodic coefficients play a special role in the understanding of the behavior of the solutions. Also other spaces than (25) can be taken to show the existence of solutions, e.g. $X = C([0,T]; C_b(\mathbb{R}^d))$.

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Control of multiscale systems with constraints
3. Geometrodynamics of the evolution of systems with varying constraints

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Abstract. With the use of the general variational principle of self-organization of systems with varying constraints, namely *the principle of dynamical harmonization of systems* presented in the first work of the cycle, we advance an approach to the control over the evolution of systems of many particles. The geometric nature of this principle is analyzed. On the basis of the de Broglie–Bohm representation of the Schrödinger equation, we establish a connection of the nonlocality and the coherence of the systems of many particles with mass entropic forces. The defining role of a coherent acceleration and a space-time curvature in the control over the synthesis of new structures in systems with varying constraints is demonstrated. The basic criteria for electromagnetic fields to initiate the processes of self-organizing synthesis and for the quantum properties of a nonlocality on macroscopic scales, which are necessary for the self-organizing synthesis, are formulated.

Contents

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1 Introductions

This work is a sequel of the cycle of works (see [1–2]), where some approach to the control over the evolution of the systems of many particles on the basis of the general variational principle of self-organization (*the principle of dynamical harmonization of systems*) is presented. The purpose of the cycle is the development of foundations of the theory and the technology of the synthesis of final states of a system of particles with desired structure and energy binding, which are attained from a given initial state with the help of the initiation of a natural evolution and the control over an evolutionary trajectory of the system at the expense of its internal power resources at a minimal use of the energy of external drivers.l

The purpose of the present work is the determination of criteria of the initiation of the self-organizing synthesis, classification of needed drivers, and development of the theory of control over the processes of synthesis on the basis of using the geometric nature of the evolution in the frame of the variational principle of dynamical harmonization.

As is known, the variational principles are the most general and brief means to formulate the laws of the Nature. For example, the equations of dynamics of a system of particles follow under very general conditions from the Gauss least-compulsion principle [3], and the equations of Maxwell and Einstein can be derived from the principle of least action [4].

Our purpose requires us to solve a strongly nonlinear optimization problem. In this problem, it is necessary, in fact, to optimize the trajectory of a system and to appropriately modify the conditions of optimization of this trajectory. Here, we will substantite a possibility of the power-informational control over the evolution of an ensemble of many particles in a noninertial reference system.

In the case under consideration, the essential point is a control nonlinearity related to the fact that the evolution of a system of particles (changes of its structure and the energy of constraints) and the space-time metric are mutually dependent. The theory of evolution of the systems with regard for their internal and external geometries, which will be developed in the present work, can be called the geometrodynamics of the evolution of the systems with varying constraints.

The principle of dynamical harmonization [1, 5] asserts that the self-organization of a system of particles, being under the action of mass forces leading to coherent accelerations of all particles of the system, is directed always to the realization of the transition from the initial state to a state with maximally free dynamics by means of changes of the structure of the system and its inertia relative to mass forces.

According to the Gauss least-compulsion principle, we should vary the accelerations of particles at a fixed velocity lying in a plane tangent to the trajectory. Hertz noticed that the varied accelerations can be related to inertial forces (mass forces) and showed for some simple cases that the minimum of the Gauss compulsion function is equivalent to the minimum of the curvature of the trajectory of a particle. This look at one of the most general variational principles of mechanics allowed one to develop the geometric interpretation for it: the trajectories of particles are geodesic lines in a space.

The idea of the geometrization of the laws of physics was intensively developed in the last century. The most remarkable example of the geometrization of physics is Einstein's general relativity theory, which established the continuous connection of geometry and matter. The fields of gravitation (it is a mass force) induce coherent accelerations and form a curvature of the space-time, where the particles are moving freely and are simultaneously the sources of the curvature of this space-time. In other words [6], "matter tells space how to curve, and space tells matter how to move." It is of importance that such an approach to the field theory allowed one not only to describe the fields of gravitation, but also to deduce the equations of motion of particles directly from the field equations [7,8], if the idea of particles as the singular solutions of the field equations is used.

The idea of particles related to singularities was somewhat earlier introduced by L. de Broglie, who tried to interpret the quantum-mechanical dynamics of particles in the frame of his theory of double solution [10–11] on the basis of the Madelung hydrodynamic representation [9] for the Schrödinger equation. In this quantum-mechanical theory, it is proposed to represent the dynamics of a particle by the sum of two solutions of the Schrödinger wave equation, namely the smooth and singular ones.

The importance of the notion of nonlocality for the theory of self-organizaltion was indicated in the first work of the cycle [1]. Here, we will refine the connection of the property of nonlocality of the wave functions determined from the Schrödinger equation (see [12]) with mass and entropic forces and will show that this property is also revealed in classical physics as a result of the geometrization of the physical processes of dynamics and evolution.

In the frame of classical physics, the geometrization of the dynamics of particles, which is garmonically associated with the property of nonlocality, was first realized by A. Vlasov. He constructed a nonlocal statistical theory [13–15] and obtained kinetic equations on the basis of the geometry of a space of support elements. "The space of support elements" includes the following notions:

1) Coordinate space.
2) Tangent space and the tangency order.
3) System of vectors loaning on the tangency point and lying in the tangent space.

The space of support elements joins the coordinate space (as the space of the possible values of the centers of mass of particles) and as the space of the possible values of kinematic parameters of particles, for example, their velocities, ans also can include the vectors of accelerations of an arbitrarily large order, which depends on the tangency order.

The validity of the space of support elements consists in the exact formation of a new understanding of a particle, which is characterized by the continuum of the possible values of coordinates and velocities (and also accelerations of any order), as distinct from the classical image of a localized particle with definite values of coordinates and velocities [14].

2 Schrödinger equation and entropic forces

The most efficient apparatus for the analysis of states of the system undergoing coherent accelerations is presented by the Schrödinger equation and covariant kinetic equations.

The Schrödinger equation for the wave function $\Psi(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N, t)$ describing the ensemble of N particles,

$$i\hbar \frac{\partial \Psi(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N, t)}{\partial t} = -\frac{\hbar^2}{2m} \sum_{n=1}^{N} \nabla_n^2 \Psi(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N, t) + \\ + U(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N)\, \Psi(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N, t), \qquad (1.1)$$

where $\nabla_n = \left\{ \frac{\partial}{\partial x_n}\vec{i} + \frac{\partial}{\partial y_n}\vec{j} + \frac{\partial}{\partial z_n}\vec{k} \right\}$, and $U(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N)$ is the potential of external forces acting on particles, arose from the attempt to solve some problems of the dynamics of particles on small spatial scales. The linear equation for complex-valued wave functions, which was obtained as a generalization of the classical dynamics of particles characterized by real variables, has shown a very good agreement with experiment. The transition from complex-valued variables in the Schrödinger equation conversely to real variables in the frame of the de Broglie–Bohm representation (see [9, 11, 16]),

$$\Psi(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N, t) = \sqrt{\rho(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N, t)} \cdot \exp\left(i \frac{J(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N, t)}{\hbar} \right), \quad (1.2)$$

where ρ—probability density, and J—action, allowed one to show that the differences between classical and quantum mechanics are reduced to the appearance of an additional potential $U_q(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N)$ in the system of equations for real variables. It was called the "quantum" potential (see the details in Section 2).

In the present work on the basis of the representation of de Broglie and Bohm for the wave function of a system of particles with regard for the potential U_q, we will obtain the equations for the entropy and the formulas for coherent accelerations in electromagnetic fields. We will analyze the solutions of kinetic equations with regard for quantum statistics for an ensemble of particles undergoing the coherent acceleration and obtain the solutions of kinetic covariant equations in noninertial reference systems.

Recently, the more and more attention was paid to the macroscopic objects possessing some quantum properties (in particular, the properties of coherence and nonlocality). In 2001, the Novel Prize in physics was awarded for the creation and study of the Bose-condensates of atomic complexes [17]. In view of the importance of the notion of a "macroscopic quantum object" (MQO), which is considered as a set of particles forming a collective system of macroscopic sizes and possesses the property of nonlocality typical of quantum objects.

The basic property of the evolution turns out to be the space-time anisoltropy related to the fact that the factor defining the evolution is an acceleration, and the properties of the space in the directions along the acceleration and perpendicularly to it are obviously different. Such an anisotropy corresponds to the obtained solutions of covariant equations that have power asymptotics and define the fast localization of the domain of existence of the system in the direction of action of a mass force and its fast delocalization in a subspace orthogonal to the direction of the acceleration.

In this case, the natural geometry of the space-time is the geometry of the space of support elements (the support elements are the kinematic elements on the trajectory of a particle), namely the Finsler geometry. Such a viewpoint combines, in fact, the approaches of L. de Broglie, J.-P. Vigier, A. Einstein, and A. Vlasov concerning the geometric nonlinear nature of the physical laws of dynamics and evolution. The role of a support is played by the four-dimensional space-time. The tangent bundles are the planes of accelerations of all orders and the space of the internal structure of a system, i.e., the space with coordinates characterizing a structure of constraints in the system (e.g., such as the fractal dimension or the order parameter, deformations, etc.). The tangent bundles and the support, which is a space-time with Riemann geometry, are coupled by the vector of acceleration or the space-time curvature.l

As for the relationship of a self-organizing system and the space-time, we note that

- the systems of particles, by undergoing the coherent accelerations during the evolution, make the space-time, where they are placed, curved;
- in turn, the space-time becoming curved indicates the directions of free motion for particles and the directions of evolution of the internal structure of the system.

The obtained solutions of covariant kinetic equations under a fast extension of the space-time becoming curved are similar to the accelerated cosmological extension in the general relativity theory (see [18]) with cosmological constant. As became clear in the 1960s [19–21], such an extension is continuously connected with the physical vacuum, which has the antigravity properties corresponding to the cosmological term in the Einstein equation.

The physical vacuum has many significant properties. In particular, it was established that vacuum is homogeneous on scales from centimeters or meters up to cosmic scales. On the "ordinary" and subatomic scales, the homogeneity of vacuum can be broken with the appearance of experimentally observed macroscopic effects related to the polarization of vacuum (Casimir effect) and to the coherent acceleration (dynamical Casimir effect) [22–24]. Near (and inside) the systems that undergo phase transitions and coherent accelerations, the space-time becomes curved (see the Vlasov theory [13–14]), and the light velocity is changed (see experimental results in [25]).

Here, we consider the possibilities to use the electromagnetic fields and the fields of entropy gradients for the control over the evolution of a system, i.e., over the evolution of its constraints. In this case, the space-time curvature arises, and, hence, the resonances of mass forces producing a inhomogeneity of the space-time and the physical vacuum can appear. These resonances are analyzed, and their parameters are determined.

The application of these resonances to the control over the synthesis of systems with varying constraints can become a promising element of the future technologies with "guided evolution".

The natural consequence of the space-time curvature is the difference of the intrinsic time from the laboratory one. The former depends not on the velocity of the reference system, but on its acceleration, which leads to a change in the lifetime of particles [26].

Similar effects were noticed by Vlasov [14] and Kozyrev [27], and the influence of the growth of crystals on the light velocity was experimentally discovered as early as 1905 [25].

It is especially interesting that Kozyrev was able, starting from his theory of time, to fabricate a special gage on the basis of resistors forming a bridge scheme and to observe the motion of stars at a laboratory on the Earth in real time (see, e.g., [28]). In our opinion, these effects are related to the appearance of a local space-time curvature determining the ratio of the intrinsic and laboratory times, as well as the impedance of electrotechnical elements (see, le.g., [29]).

The close ideas were developed in works by S. Podosenov (see [29]), where he showed how the constraints in a system determine the Riemann space-time curvature.

In the present work, we will show that the variational principle of dynamical harmonization leads to the geometrization of the physical processes of evolution and to a generalization of the theory for any fields of mass forces. We give the experimental results obtained with the use of the Kozyrev detector. We consider that they demonstrate, under laboratory conditions, the resonances of electromagnetic radiation with the inhomogeneity related to the space-time curvature initiated by the action of electromagnetic drivers described below.

The resonances caused by the propagation of longitudinal waves amplify the fluctuations of vacuum under conditions of the scale invariance and, acting on particles, initiate their complicated motion, which can develop into a dynamical chaos. As was shown in [30], it is convenient to apply Tikhonov's methods of regularization of the states with dynamical chaos to the description of the dynamics of particles. The scale invariance of vacuum implies that the operators of regularization can be given in terms of quantum [31] and fractional [32] integro-differential operators. The developed model of the interaction with vacuum describes naturally the openness of the system and the processes of creation and decay [33].

The equations of the dynamics of particles at their interaction with the scale-invariant vacuum after the regularization can be modeled with electrophysical curcuits: operational amplifiers as a model of the operators of regularization and branched equivalent resonance schemes as a model of fractal medium. The developed model is especially useful for the optimization of electromagnetic drivers.

We emphasize once more that a specific feature of the considered technology of the control over the evolution of systems is the use of the internal mass-defect energy in order to change a structure of the system, rather than the energy of external drivers. The low energy of external fields must be spent only for the control and the initiation of the processes of self-organization with desired structural and energetic directedness. In what follows, we will show that namely the nonlocal entropic fields determine the main properties of MQO.

2.1 The de Broglie–Bohm representation for the Schrödinger equation and entropic forces

The main property of MQO (i.e., a quantum system with constraints) consists in that the wave function describing it cannot be represented as a product of one-particle wave functions in the form

$$\Psi\left(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N, t\right) = \Psi\left(\vec{r}_1, t\right) \Psi\left(\vec{r}_2, t\right) \ldots \Psi\left(\vec{r}_N, t\right). \tag{2.1}$$

For this function, the normalizing integral is reduced to product of independent integrals:

$$\int\limits_{V_1} \Psi^*\left(\vec{r}_1, t\right) \Psi\left(\vec{r}_1, t\right) dr_1 \cdot \ldots \cdot \int\limits_{V_N} \Psi^*\left(\vec{r}_N, t\right) \Psi\left(\vec{r}_N, t\right) dr_1 = 1. \tag{2.2}$$

Each integral in this expression is separately equal to 1. In other words, the behavior of each particle in the ensemble is described by the own wave function independent of the states with other wave functions. Hence, the coupling of particles in the single ensemble is ensured only due to the potential of external fields $U\left(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N\right)$ and to the quantum potential $U_q\left(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N\right)$ [34]. This coupling is supported by quanta ensuring the given interaction. The rate of exchange by quanta does not exceed the light velocity, as distinct from the entangled states representing MQO, where the information propagates, as will be shown below, instantly. This can be easily seen from the formula for the probability to find MQO in a given volume V:

$$\int\limits_V \Psi(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N, t)^* \Psi\left(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N, t\right) dr_1 dr_2 \cdots dr_N = 1. \tag{2.3}$$

In this case, the wave function describing MQO depends on the time, but the probability to find MQO in the given volume is conserved. This implies that any change in the position of any of N particles placed in a volume V affects instantly the positions of all remaining bound particles.

Thus, the instant rearrangement of the positions of all particles reflects the presence of strong constraints in MQO and, hence, strong correlations in it. MQO is a macroscopic formation (most probably, a quasicrystal), which is described by the quantum Schrödinger equation.

A bound dynamical system tends to instantly become self-organized and to pass in a state with maximal probability. For the viewpoint of the principle of dynamical harmonization, the system chooses optimally the direction of a change of the structure formed by entropic fields. This reasoning can be easily generalized to the case of a partially bound dynamical quantum system.

With regard for the de Broglie–Bohm representation (1.2), Eq. (1.1) yields

$$
\begin{aligned}
-\frac{\partial J}{\partial t}\Psi + i\hbar\frac{1}{2\rho}\frac{\partial \rho}{\partial t}\Psi = {} & \frac{1}{2m}\sum_{n=1}^{N}\left(\nabla_n J\right)^2\Psi + U\left(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_N\right)\Psi - \\
& -\frac{i\hbar}{2m}\sum_{n=1}^{N}\nabla_n^2 J\cdot\Psi - \frac{i\hbar}{2m}\sum_{n=1}^{N}\left(\frac{\nabla_n\rho}{\rho}\right)\left(\nabla_n J\right)\cdot\Psi - \\
& -\frac{\hbar^2}{2m}\sum_{n=1}^{N}\left(\frac{\nabla_n^2\rho}{2\rho}\right)\Psi + \frac{\hbar^2}{2m}\sum_{n=1}^{N}\left(\frac{\nabla_n\rho}{2\rho}\right)^2\Psi. \qquad (2.4)
\end{aligned}
$$

We note that the probability density and the action are real functions. We can separately collect the real and imaginary terms and obtain two nonlinear equations corresponding to one linear Schrödinger equation:

$$
\begin{aligned}
-\frac{\partial J}{\partial t} = {} & \frac{1}{2m}\sum_{n=1}^{N}\left(\nabla_n J\right)^2 + U\left(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_N\right) + \\
& + \frac{\hbar^2}{2m}\sum_{n=1}^{N}\left(\left(\frac{\nabla_n\rho}{2\rho}\right)^2 - \frac{\nabla_n^2\rho}{2\rho}\right), \qquad (2.5)
\end{aligned}
$$

$$
-\frac{\partial \rho}{\partial t} = \sum_{n=1}^{N}\nabla_n\cdot\left(\frac{\rho\nabla_n J}{m}\right). \qquad (2.6)
$$

We now introduce the entropy density of a quantum system in the form

$$
\begin{aligned}
S\left(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_N,t\right) &= -\ln\left|\Psi\left(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_N,t\right)\right|^2 \\
&= -\ln\left(\rho\left(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_N,t\right)\right), \qquad (2.7)
\end{aligned}
$$

Let us transform formulas (2.5) and (2.6), by substituting entropy (2.7) in them. In this case, we take into account that $\rho^{-1}\nabla\rho = \nabla\ln\rho$. We have

$$
\begin{aligned}
-\frac{\partial J}{\partial t} = {} & \frac{1}{2m}\sum_{n=1}^{N}\left(\nabla_n J\right)^2 - \frac{\hbar^2}{8m}\sum_{n=1}^{N}\left(\nabla_n S\right)^2 + \\
& + U\left(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_N\right) + \frac{\hbar^2}{4m}\sum_{n=1}^{N}\nabla_n^2 S. \qquad (2.8)
\end{aligned}
$$

We note that the modified Hamilton–Jacobi equation (2.8), which passes into the classical Hamilton-Jacobi equation at the standard limiting transition as $\hbar \to 0$, contains the term $\frac{\hbar^2}{8m} \sum\limits_{n=1}^{N} (\nabla_n S)^2 = E_s$. This term is an analog of the entropic-type kinetic energy E_s and is expressed through the gradients of entropic fields. The physical meaning of a kinetic energy of the entropic type consists in the binding of a system of particles, which causes a decrease of their total kinetic energy. The action of the entropic force is directed on the optimization of the system of constraints in dynamical system, by leading it to the most probable state. In other words, the system evolves to a new configurational state with maximal stability. The density gradients of entropic fields are a quantitative characteristic of the probabilistic laws and forces.

In the modified Hamilton–Jacobi equation (2.8), the last term $\frac{\hbar^2}{4m} \sum\limits_{n=1}^{N} \nabla_n^2 S$ corrects the potential energy of the system. The sign of the Laplace operator in this expression reflects a decrease or increase in the potential energy of the bound system on the whole. Thus, the direction of the flows of entropy density gradients determines the final value and the shape of the potential of interaction between particles.

Thus, it becomes clear that the entropic field is related to the fields of constraints in any quantum system, in particular, in MQO. Moreover, the introduction of entropic forces makes the separation of quantum and classical mechanics, which has born always a sufficiently indefinite character, to be conditional. The border between them is eroded, if we consider the dynamics of classical systems with regard for the evolution of their internal constraints in the presence of the corresponding entropic fields.

From our viewpoint, it is significant that formula (2.8) contains quantum terms together with classical ones. The former can be expressed in terms of the operators of momentum and kinetic energy of particles of the system: $\widehat{p}_n = -i\hbar\nabla_n$, $\widehat{T}_n = -\frac{\hbar^2}{2m}\nabla_n^2$.

Using these operators, we can transform Eq. (2.7) to the form

$$-\frac{\partial J}{\partial t} = \frac{1}{2m}\sum_{n=1}^{N} (\nabla_n J)^2 + \frac{1}{8m}\sum_{n=1}^{N} (\widehat{p}_n S)^2 + + U(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N) - \frac{1}{2}\sum_{n=1}^{N} \widehat{T}_n S. \tag{2.9}$$

Formulas of the type (2.10), which include classical terms and the operators of physical quantities, describe the macroscopic quantum objects. Thus, MQOs reveal both classical and quantum properties.

The equation of balance of the entropy follows simply from Eq. (2.6):

$$\frac{\partial S}{\partial t} + \sum_{n=1}^{N} (\vec{u}_n \cdot \nabla_n S) - \frac{1}{m}\sum_{n=1}^{N} \nabla_n^2 J = 0. \tag{2.10}$$

The knowledge of solutions of the system of nonlinear differential equations (2.8), (2.10) for the action and the entropy allows us to write the wave function, being a solution of the Schrödinger equation, in the form

$$\Psi\left(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N, t\right) = \exp\left(-Z\right),$$
$$Z = \frac{S\left(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N, t\right)}{2k} - i\frac{J\left(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N, t\right)}{\hbar}. \tag{2.11}$$

Here, we write the Boltzmann entropy in the form $S = -k_B \ln \rho$, where k_B is the Boltzmann constant. The function Z is a complex-valued function with nonzero real and imaginary parts.

Let us consider the action of the operators of momentum and kinetic energy on the entropy:

$$\widehat{p}_n S = \frac{1}{8m}\left(\frac{1}{\Psi^*}\left(\widehat{p}_n \Psi^*\right) - \frac{1}{\Psi}\left(\widehat{p}_n \Psi\right)\right). \tag{2.12}$$

If the eigenvalues of the operator of momentum are real numbers, then the action of the operator of momentum on the entropy is equal to zero, but it is possible only in the case where the wave function of the system of particles can be expanded in a product of wave functions.

The action of the operator of kinetic energy on the entropy is given as follows:

$$-\frac{1}{2}\sum_{n=1}^{N}\widehat{T_n}S = \sum_{n=1}^{N}E_n - \sum_{n=1}^{N}\frac{p_n^2}{2m} = E - \frac{P^2}{2m}. \tag{2.13}$$

In this case, the eigenvalues of the operator of kinetic energy are real. We denote such an eigenvalue for the n-th particle as E_n and the total momentum of particles of the system as P.

It is seen from (2.13) that the action of the operator of kinetic energy on the entropy of the system is since $E = \dfrac{P^2}{2m}$. This result is possible only under the condition that the wave function of the system of particles can be expanded in a product of one-particle wave functions, which breaks MQO. In this case, the generalized quantum-classical Hamilton-Jacobi equation passes into its classical analog.

The performed calculations allow us to assert that **the quantum corrections related to the entropic fields appear only in the presence of long-range correlations in the systems of particles, i.e., if MQO arises.**

By analogy with classical mechanics, it is easy to determine the momenta that are determined by the mass entropic fields. As is seen from the formula for the entropic-type kinetic energy, each entropic momentum p_{sn} acquired by the n-th particle is proportional to the entropy gradient.

$$p_{sn} = \frac{\hbar}{2}\nabla_n S. \tag{2.14}$$

The entropic momentum transfers each of the particles of MQO in the position that corresponds to the maximum of the probability of a state for the given MQO. Thus, the dominating mass force (general dominating perturbation) [5] supplies coherently the momentum p_{sn} of a directed motion to all elements of the ensemble of particles, which meets the condition $\left|\sum_{n=1}^{N} \vec{p}_{sn}\right| \gg P_{T_{\max}}$, where $P_{T_{\max}}$ is the maximal absolute value of the momentum of the intrinsic heat motion of any of the elements of the ensemble (MQO, in this case).

The principle of dynamical harmonization [1] implies that the evolution of a self-organizing system is possible only in the presence of the coherent acceleration of the entire system, when all particles of the system acquire the same momentum increment due to the action of the entropic force arising under the nonzero entropy gradient. In this case, it is necessary that the regular component of the change in the momenta of particles Δp_S at the expense of the entropy exceed the chaotic heat component $p_T \approx m\, u_T$. We call this requirement as the condition of domination of a driver.

It is convenient to introduce the coefficient of domination of a driver α_d as the ratio of the momentum increment of a particle due to the action of a driver to the heat component of the momentum. Then the condition of domination of a driver takes the form

$$\alpha_d \gg 1, \qquad \alpha_d = \frac{\Delta p}{p_T}. \tag{2.15}$$

The rate of transfer of a momentum to the system of particles allows us to estimate the mass force F_{str} stimulating the system to the coherent acceleration and the evolution due to a change of the internal structure.

Thus, the analysis of the Schrödinger equation implies that the perturbation of MQO related to the appearance of the entropy gradient (mass force) at any point of the volume occupied by MQO causes a change of the momentum of each particle of the given object, since even an insignificant external entropic perturbation acts at once on all particles, which are located at the points $\{\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N\}$.

Such properties of the system indicate the existence of a nonlocality of MQO in the general case. It becomes clear that the physical entropic field (field of mass forces) is the reason for the appearance of the fields of constraints.

If condition (2.15) is satisfied, and if the system undergoes the action of an entropic dominating perturbation (i.e., if $\nabla S > 2P_{T_{\max}}/\hbar$ is satisfied), the value of "quantum potential" becomes essential for the evolution of the system irrespective of its scale. In other words, under the action of a dominating perturbation, even a classical system becomes nonlocal and acquires quantum properties.

It is seen from Eq. (2.13) that the entropic fields acting on the system of particles decrease always its kinetic energy. For such systems, we introduce the definition of the degree of nonideality Θ, which shows a share of the decrease of constraints of the system:

$$\Theta = \frac{E - E_s}{E}. \tag{2.16}$$

Here,

$$E = \frac{1}{2m}\sum_{n=1}^{N} (\nabla_n J)^2 = \frac{1}{2m}\sum_{n=1}^{N} p_n^2$$

is the kinetic energy of the system of N particles with the same mass, and $\vec{p}_n = \nabla_n J$.

For $\Theta = 0$, the system is completely bound, has the maximal ideality, and possesses, as a whole, a field of entropy gradients such that leads to a minimum of the kinetic energy. Based on this, we can estimate the mean entropy gradient, due to which the ideality limit and the maximal coherence are attained in the system, i.e., $E \approx E_s$ or $\langle p_s \rangle \approx \langle p \rangle$:

$$\langle \nabla S \rangle \approx \frac{2}{\hbar} \langle p \rangle . \tag{2.17}$$

For $\Theta = 1$, the fields of entropy gradients are absent, and the system becomes completely nonideal.

The introduction of the entropic momenta (2.14) leads to a new formula for entropic forces in quantum mechanics,

$$\vec{F}_s = \dot{\vec{p}}_s = \frac{\hbar}{2} \nabla \dot{S}, \tag{2.18}$$

where the dot stands for the differentiation with respect to the time. Hence, the entropic force is proportional to the entropy production gradient $\sigma_S = \dot{S}$ (see also [1]) in the system. This formula differs from that obtained by E. Verlinde [35], who used the equations of equilibrium thermodynamics of closed systems and obtained

$$\vec{F}_s = T \nabla S. \tag{2.19}$$

This formula does not involve the entropy production and, hence, cannot be applied to the description of nonequilibrium systems.

Knowing the entropic force, it is easy to set the entropic pressure into the theory:

$$P_K = \frac{(F_s, \vec{n})}{K}, \tag{2.20}$$

where K is the area on which the entropic force acts, $\vec{n}$ is a unit vector of normal to the surface with area K. The physical meaning of the entropic pressure is that it forms structures at all hierarchical levels of the system (clusters, molecules, atoms, nuclei, etc.) by changing the space-time structure of the physical vacuum. In the case of a spherical surface with a radius R, the entropic pressure equals:

$$P_K = \frac{\hbar}{2} \frac{\nabla \dot{S}}{4\pi R^2}. \tag{2.21}$$

In order to estimate the entropic pressure in the shell with the thickness d the following formula may apply:

$$\nabla \dot{S} \approx \frac{1}{d} \left(\frac{\Delta S}{\tau} \right), \qquad P_K \approx \frac{\hbar}{2} \frac{\Delta S}{4\pi R^2 d} \frac{1}{\tau}, \tag{2.22}$$

where τ is the time for the formation of a spherical shell structure at all levels of the hierarchy, ΔS is the entropy change in the spherical shell of radius R and thickness d.

Taking into account that the entropic pressure has the same value at all levels of the hierarchy, one can determine the change of energy of constrains of the nuclear component of the system. For this to be done it is necessary to equate the work done by the entropic force on the structuring of a spherical shell at nuclear level and the change of the energy of constrains:

$$P_K dV = d\varepsilon, \qquad dV = 4\pi R^2 dR. \tag{2.23}$$

Thus, the change of the energy of constrains is proportional to the production of entropy:

$$d\varepsilon = \frac{\hbar}{2} d\left(\sigma_S\right), \qquad \sigma_S = \dot{S}. \tag{2.24}$$

and for the estimation of change of energy of constrains of the system, a relation may be applied:

$$\Delta\varepsilon = \frac{\hbar}{2}\frac{\Delta S}{\tau}. \tag{2.25}$$

This estimate provides a basis to generalize the Heisenberg uncertainty principle for energy and time in systems with varying constrains (due to the change of energy of ΔE):

$$\Delta t \Delta E \approx \frac{\hbar}{2}\Delta S. \tag{2.26}$$

Now it is clear that the transition to classical mechanics does not occur while $\hbar \to 0$, which is not actually logical for the constant, but through vanishing of entropy gradient cnange in the system. We now estimate the mass forces and the domination of a driver in the frame of the "shell" theory of evolution (see [1]). Let the action of a driver on the system of particles have lead to the separation of a subsystem of particles (shell) with mass number A_{sh}, where the mass force acts.

The coefficient of domination α_{dS} of the action of an entropic driver on a single particle can be represented as

$$\begin{aligned}\alpha_{dS} = \frac{\hbar\left(\nabla S/A_{sh}\right)}{m_p} &= \left(\frac{\hbar}{m_p}\right)\frac{\left(\Delta S/A_{sh}\right)}{u_T l_{eff}} \\ &\approx 10\left(\frac{\hbar/\left(m_p u_T\right)}{R_{sh}}\right)\Delta S \approx 10\left(\frac{\lambda\!\!\!^{-}_{D-B}}{R_{sh}}\right)\frac{\Delta S}{A_{sh}},\end{aligned} \tag{2.27}$$

where $\lambda\!\!\!^{-}_{D-B}$ is the de Broglie wavelength corresponding to a heat pulse:

$$\lambda\!\!\!^{-}_{D-B} \approx \hbar/p_T. \tag{2.28}$$

To obtain the final result, we need to estimate a change of the entropy $(\Delta S)_b$ due to the development of physical processes with a change of constraints in the system. On the initial stage, a subsystem of A_{sh} particles is separated. In it, the initial shell structure with the coherent part of A_{shCog} particles and, hence, with the input order parameter $\eta_{sh} = \dfrac{A_{shCog}}{A_{sh}}$ is created. Then we take into account that

- the break or creation of one constraint consumes the erasure energy of one bit of information $\varepsilon_b \approx T \ln 2$ (Landauer theorem [36]);
- the number of constraints at the formation of a cluster with with mass number A_{shCog} is equal, by the order of magnitude, to $N_{bcog} \approx 0.5(A_{shCog})^2$;
- the probability of the creation of a cluster with the number of particles A_{shCog} is proportional to $P_{cog} \approx 1/\sqrt{A_{shCog}}$;

- $A_{shCog} \approx \eta_{sh} A_{sh} Z$;
- $T(\Delta S)_b = W_{shCog} Z$, $V \approx const$, W_{shCog} is the energy of the formation of the coherent part of a shell;
- $W_{sh} \approx P_{cog} N_{bcog} \varepsilon_b Z$.

The above relations imply that, at the formation of a structure, the entropy is changed by

$$(\Delta S)_b \approx \frac{1}{T} W_{shCog} \approx \frac{1}{T} 0.5 (A_{shCog})^2 \frac{1}{\sqrt{A_{shCog}}} T \ln 2$$
$$\approx \frac{\ln 2}{2} (A_{shCog})^{3/2} \approx 0.038 (\eta_{sh} A_{sh})^{3/2}. \tag{2.29}$$

In this case, the specific change of the entropy per particle is

$$\frac{(\Delta S)_b}{A_{sh}} \approx 3.8\, 10^{-2} \eta_{sh}^{3/2} A_{sh}^{1/2}. \tag{2.30}$$

Substituting this formula in that for the coefficient of domination (2.27), we obtain

$$\alpha_{dS} \approx 0.38 \left(\frac{\lambda\!\!\!^{-}_{D-B}}{R_{sh}} \right) \eta_{sh}^{3/2} A_{sh}^{1/2}. \tag{2.31}$$

If the conditions of domination (2.14) are satisfied, the drivers transfer the nucleus of a shell from the quasiequilibrium state in the inertial reference system in a strongly nonequilibrium state in the noninertial system formed under the action of mass forces $F_{str} = \frac{d\Delta p}{dt}$. *The mass forces cause the coherent acceleration*

$$a_{cog} \approx \frac{1}{m_p} \frac{d}{dt} (\Delta p) \approx \frac{1}{m_p} \left(\frac{\alpha_d p_T}{\tau_{eff}} \right) \approx \alpha_d \frac{u_T}{\tau_{eff}} \tag{2.32}$$

and the evolution of a structure of the system (see the principle of dynamical harmonization [1]).

We note that the basic relations have been obtained from the Schrödinger equation in the nonrelativistic case without external electromagnetic field.

2.2 Electromagnetic drivers of mass forces

We now show that the electromagnetic fields can be dominating perturbations for a system of particles. As is well known, the Lagrange function of a system of particles in electromagnetic fields is transformed ([37]). Without electromagnetic fields, the momentum of a particle with charge q and mass m is connected with its velocity $\vec{u}$ by the well-known formula $\vec{p} = \frac{m\vec{u}}{\sqrt{1-(u/c)^2}}$. At the motion in an electromagnetic field with the vector potential $\vec{A}$ (and in the fields defined by the potential: $\vec{B} = rot\,\vec{A}$, $\vec{E} \propto \frac{\partial \vec{A}}{\partial t}$), the total momentum of the particle changes due to the vector potential. Even if there is no magnetic field at the point of the space, where a particle is located, the total momentum of the particle is determined by the formula

$$\vec{P} = \frac{m\vec{u}}{\sqrt{1-(u/c)^2}} + e\vec{A}. \qquad (2.33)$$

Similarly to the connection of the electrostatic potential and the energy, the vector potential reveals a connection with the momentum. The vector potential supplies the additional electrodynamic momentum to all charged particles $\Delta\vec{p}_{EM} = e\vec{A}$.

The mass force of the electromagnetic origin, $F_{str} = \frac{d(\Delta p)_A}{dt} = q\frac{dA}{dt}$, is a force acting on a charge and is given by the derivative of the vector potential with respect to the time in the standard formula for the electric field intensity in terms of the four-dimensional gradient of a four-dimensional potential:

$$\vec{E} = -\nabla\varphi - \frac{\partial\vec{A}}{\partial t}. \qquad (2.34)$$

We note that the contribution of the rate of variation of the vector potential

- can essentially exceed that of the electrostatic potential gradient for short pulses;
- can be present in the system even without gradient electric fields and transverse magnetic fields ($rot\vec{A} \approx 0$, and the potential is frequently called a zero-field potential in this case);
- by determining an alternating electric field $\vec{E}$ if the condition $rot\vec{A} \approx 0$ holds, generates no alternating magnetic field, but can ensure the appearance of sources of a vector potential that are concentrated in the regions with $div\vec{A} \neq 0$;
- defines the localization of the magnetic field and sources of a vector potential in spatially remore regions;
- can be present in the system in the case where the electrostatic potential is the same at all points of the system.

There are many means to generate the fields of a vector potential, but such sources as the toroidal coils on a core with magnetic permeability μ and with current I_{amper} flowing on n windings are most convenient. For such drivers, the amplitude of the vector potential is given by the relation

$$A \approx \frac{\mu_0}{4\pi}\mu n\, I_{Amper} = 10^{-7}\mu n\, I_{Amper}$$

(in IS units) and ensures the coefficient of domination

$$\alpha_{dA} = \frac{qA}{m_p u_T} = 10^{-7}\mu\cdot n\cdot\left(\frac{q}{m_p}\right)\frac{I_{Amper}}{u_T}. \qquad (2.35)$$

In correspondence with the equations Maxwell, the component of the electric field intensity $\vec{E} = -\frac{\partial\vec{A}}{\partial t}$ exists also in a homogeneous system of particles, i.e., it can be an electromagnetic mass force acting directly on the charged component of a system of particles.

We now mention one of the simplest drivers. The collective properties of a system of particles are usually revealed, first of all, in the hydrodynamic behavior of the system. It is clear that if the system is affected by a hydraulic impact, then, at its high intensity, the particles receive a momentum increment $\Delta\vec{p}_m$ exceeding the thermal momentum. Moreover, a nonlinear wave moving with supersound velocity appears in the system of particles. It is clear that, in this case, the condition of domination is satisfied on the front of this nonlinear wave is satisfied.

We now summarize the above-performed analysis of the Schrödinger equation: the particles undergo the action of a nonlocal mass force causing a change in the momentum of particles by a value bounded by the sum of the corresponding contributions of basic drivers (mechanical, electromagnetic, and entropic ones): $|\Delta p| \leqslant |\Delta\vec{p}_m| + |\Delta\vec{p}_A| + |\Delta\vec{p}_S|$.

Here, we consider the following main channels of transfer of a momentum to particles:

- macroscopic hydrodynamic impact leading to increments of the momenta of the particles (the impact can be realized, in particular, by longitudinal acoustic waves in a medium)

$$\Delta p_m \approx m\Delta u_{sh}; \tag{2.36}$$

- direct impact increment of a momentum in the electromagnetic field

$$(\Delta p)_A \approx qA \tag{2.37}$$

  (in IS units);
- increment of a momentum in the field of entropy gradients

$$(\Delta p)_S \approx \hbar\,(\nabla S)\,. \tag{2.38}$$

It is clear that the action of sources of impulsive excitation on a system of many particles leads to a nonequilibrium state. As was shown in the works of A. Vlasov [13–15], the kinetic equation for the collective states of a system of particles with distribution function $f\,(\vec{r},\vec{u},t)$ can be presented in a closed divergent form in the Finsler space.

Usually even in the case of a strong deviation from the equilibrium, the system is described within the Prigogine method of locally equilibrium distributions [38] with parameters depending on the spatial coordinates. In this case, the kinetic theory describes the evolution of both a state of the system of particles and the distribution of particles in the configurational space. In this case, the sources are usually positioned on boundary of the region under consideration and act on different particles of the system differently. *The main parameter of a driver defining the kinetics is the power flow density on the boundary of the system.*

In the cases where a driver initiates the action of a mass force on the system, it renders, by definition, the practically identical action on all particles irrespective of their location in the system. In other words ρ particles in unit volume receive the same momentum increment $\Delta\vec{p}$ for the time Δt. *Therefore, the parameter defining the openness of the system is, naturally, the bulk density of a power absorbed in the system,* $P_A = \nu_{eff}\rho_W$.

In the case of the action of a nonstationary vector potential, the relations $E \approx \nu_{eff} A$ and $\rho_W \approx (\nu_{eff} A)^2$ are satisfied, and we have

$$P_A \approx \nu_{eff} |E_A|^2 \approx \nu_{eff}^3 A_{eff}^2 \approx \frac{A_{eff}^2}{\tau_{eff}^3}. \qquad (2.39)$$

It is seen that the force action efficiency increases as the third power of the frequency with the corresponding decrease in the impact duration.

We now construct a dimensionless parameter characterizing the degree of "impactness" of an action. To this end, we estimate the dissipation power P_{dis} and consider the dimensionless ratio $Q_{imp} = \dfrac{P_A}{P_{dis}}$ (impact factor or the coefficient of impactness) of the power density of the driver to the dissipation power density $P_{dis} \approx \dfrac{\rho T}{\tau_{dis}}$. In view of (2.32), the parameter of impactness

$$Q_{imp} = \frac{A^2}{\rho T} \frac{\tau_{dis}}{\tau_f^3} \approx \left(\frac{\tau_{dis}}{\tau_f} \right) \cdot \left(\frac{R_{WZ}}{r_e} \right) \cdot \left(\beta_T \frac{a}{a_{dis}} \right)^2, \qquad (2.40)$$

where $r_e = \dfrac{e^2}{m_e c^2}$ is the classical radius of an electron, and R_{WZ} is the radius of a Wigner–Seitz cell.

The distinctive property of the parameter of impactness is its reciprocal dependence on the cube of the characteristic duration of an impact action.

For the system of particles, the bulk density of a consumed power characterized by the parameter of impactness is the nonequilibrium source in the kinetic equation for the distribution function of particles, which is the equation of continuity in the space with coordinates $(\vec{r}, \vec{u})$ for the effective medium represented by the probability distribution:

$$\frac{\partial f(\vec{r}, \vec{u}, t)}{\partial t} + div_{\vec{r}} \left(\vec{u} f(\vec{r}, \vec{u}, t) \right) + div_{\vec{u}} \left(\langle \dot{\vec{u}} \rangle f(\vec{r}, \vec{u}, t) \right) = \Psi(r, p). \qquad (2.41)$$

Here, $\langle \dot{\vec{u}} \rangle = \dfrac{\int d\dot{\vec{u}}\, \dot{\vec{u}}\, f\left(\vec{r}, \vec{u}, \dot{\vec{u}}, t\right)}{f(\vec{r}, \vec{u}, t)}$ is the acceleration averaged over the whole ensemble of particles, and the distribution function is defined in the space of support elements, the Finsler space. It will be described below. We will show that the *properties of the space, where the distribution functions are defined, are of great importance and allow one to naturally describe the self-organization of the systems of particles even without explicit presence of forces of the fundamental nature.*

Consider a system of particles under the action of a dominating perturbation of mass forces without any dependence on the coordinates: $\Psi(r, p) \equiv \Psi(p)$.

It is clear that if all positions of particles in the system are equivalent for the action of a mass force, and if the condition of domination is satisfied, then the good zero approximation is a nonequilibrium system with spatially and statistically homogeneous properties, so that its principal evolution runs in the energetic conponent of the phase space.

The source or sink of energy $\Psi(p) \approx Q_{imp}\phi(p)$ (mass force), which is homogeneous in the wholespace, is characterized (by the Heisenberg uncertainty relation) by a strong localization in the momentum space. So, we may consider a linear combination of k *delta-functions as sources and sinks* concentrated near certain points of the momentum space p_k.

For the isotropic part of the distribution function, it is convenient to pass from momenta to energies with the use of the dispersion law $\varepsilon = \varepsilon(p)$ and to present the kinetic equation in the form [39–40]

$$\frac{\partial f(\varepsilon,t)}{\partial t} + \frac{1}{g(\varepsilon)}\frac{\partial}{\partial \varepsilon}\left(\Pi(\varepsilon,\{f\})\right) = \sum_k (Q_{imp})_k \frac{1}{g(\varepsilon)}\delta(\varepsilon-\varepsilon_k), \tag{2.42}$$

where we used the density of states $g(\varepsilon)$ and the flow of particles in the phase space $\Pi(\varepsilon,\{f\})$. In the Vlasov equation (2.45), the flow in the phase space corresponded to classical statistics:

$$\Pi(u,\{f\}) = \langle\dot{\vec{u}}\rangle f(\vec{r},\vec{u},t). \tag{2.43}$$

Let the particles of a substance be fermions. Taking the properties of quantum statistics into account, we will use, first of all, the fact that the mean acceleration of fermions $\langle\dot{\vec{u}}\rangle$

1) is proportional to the number of free sites for the evolution in the energy space, i.e., to the quantity $(1 - f(\vec{r},\vec{u},t))$;
2) is caused by mass forces and is determined, as is seen from the analysis of the Schrödinger equation, by the entropy $S(\{f\}) \approx \ln(f)$.

In view of this, we restrict ourselves in the expansion of the acceleration $\langle\dot{\vec{u}}\rangle$ by terms up to the first derivative $\dfrac{\partial S}{\partial \varepsilon}$. Then the flow in the kinetic equation for fermions takes the form

$$\Pi(\varepsilon,\{f\}) = \bar{a}_0(\varepsilon)\left(T_{eff} f \frac{\partial S(\{f\})}{\partial \varepsilon} + (1-f) f\right). \tag{2.44}$$

In the regions between sources and sinks, Eq. (2.42) ensures the constancy of the flows with the corresponding sign. For the zero coefficient of impactness, the solution is the Fermi–Dirac function

$$f(\varepsilon) = \frac{1}{1+\exp\left(-\dfrac{\varepsilon_F-\varepsilon}{T_{eff}}\right)}.$$

If the impactness is nonzero, the distribution function generalizes the equilibrium distribution:

$$f_q(\varepsilon) = \frac{1}{1+\exp_q\left(-\dfrac{\varepsilon_F-\varepsilon}{T_{eff}}\right)}, \tag{2.45}$$

where the parameter of nonequilibrium q is determined by the parameter of impactness, and the exponential function is replaced by the functions [36] with power asymptotics,

$$\exp_q(-x) = \left(1+\frac{q-1}{q}(-x)\right)^{\dfrac{q}{q-1}}, \qquad q = \sqrt{1+\alpha_I (Q_{imp})_k}. \tag{2.46}$$

The quantity α_I in the formula for the parameter q is determined by the information transfer rate along a communication channel between the scales and can be evaluated by the Shannon–Hartley theorem:

$$\alpha_I \approx \frac{\Delta\omega}{\omega_{eff}} \log_2 (1 + Q_{imp}) \approx \frac{\xi(t)}{Q} \log_2 (1 + Q_{imp}) . \qquad (2.47)$$

Here, $\Delta\omega$ is the transmission band of a communication channel, ω_{eff} is the effective frequency representing the action of a driver (frequency of electromagnetic signals, inverse duration of the front of pulses of the electromagnetic field, etc.), Q is the quality of the oscillatory system, and the function $\xi(t)$ represents a possible modulation of information.

Using the distribution function of particles (2.40), we can write the distribution functions $f_{bq}(\varepsilon)$ over energies for holes (antiparticles):

$$f_{bq}(\varepsilon) = 1 - f_q(\varepsilon) = \frac{\exp_q\left(-\dfrac{\varepsilon_F - \varepsilon}{T_{eff}}\right)}{1 + \exp_q\left(-\dfrac{\varepsilon_F - \varepsilon}{T_{eff}}\right)} . \qquad (2.48)$$

This formula can be used for the determination of the levels of fluctuations and excitation of the vacuum state of particles, for example, electrons and positrons.

Having defined the main parameters of drivers, we pass to the analysis of the general evolution of systems with constraints and to the clarification of its nature and mechanisms.

3 Geometrodynamics of systems with varying constraints

The main element of kinetic theory, namely the distribution function of particles, is practically the same as the probability density distribution (the squared modulus of the wave function), which is determined by solving the Schrödinger equation. However, there exists a difference between kinetics and quantum mechanics. It consists in that *the Schrödinger equation and other equations of quantum mechanics contain nonlocal terms* (which is confirmed in numerous experiments),*whereas the ordinary approaches to kinetics based on classical dynamics involve no nonlocality.*

The absence of a possibility to describe the nonlocal effects observed experimentally is the main shortcoming of practically all approaches to kinetics. A generalization of kinetics to nonlocal processes would erase these differences between classical and quantum descriptions of systems.

This purpose was attained, in the basic part, by A. Vlasov on the way of the geometrization of kinetics in the nonlocal mechanics developed by him. An analogous approach is used in our theory of evolution.

3.1 Space-time with Finsler geometry

The geometric interpretation of mass force action on objects of any nature is that the mass force creates a structure of space-time wherein the further evolution of the system occurs. Naturally, there is a need to consider the physical processes in Riemann spaces, the fiber spaces with different bases, Cartan and Finsler spaces, etc.

The geometries of Euclid and Riemann, which are usually applied to physics, concern only local properties. To describe the nonlocality of a system, A. Vlasov used the geometry of support elements (the geometry of a stratified space) [14–15], whose advantage consists in that the kinematic characteristics of the dynamics of particles become inherent internal characteristics of the system and are not imposed from outside. Any particle is characterized nonlocally, i.e., by the whole spectrum by the own geometric and kinematic properties for every time moment t: $\vec{r}, \vec{u}, \dot{\vec{u}}, \ddot{\vec{u}}, \dddot{\vec{u}}, \ldots$

The differentials of the independent coordinates, $x^0 = ct$, x^1, x^2, x^3, are infinitely small intervals basing on a current point M in the four-dimensional Riemann space-time, i.e., in the space-time with metric properties that are determined by the metric, namely the elementary interval written in terms of the differentials of coordinates of the space:

$$ds^2 = g_{ik} dx^i dx^k, \qquad i = 0,1,2,3, \quad k = 0,1,2,3. \tag{3.1}$$

Sometimes, it is convenient to separate the spatial coordinates, the time coordinate, and the spatial interval with the corresponding metric:

$$\begin{aligned} ds^2 &= g_{\alpha\beta} dx^\alpha dx^\beta + 2g_{0\alpha} dx^0 dx^\alpha + g_{00} (dx_0)^2, \\ dl^2 &= \gamma_{\alpha\beta} dx^\alpha dx^\beta, \\ \gamma_{\alpha\beta} &= -g_{\alpha\beta} + \frac{g_{0\alpha} g_{0\beta}}{g_{00}}. \end{aligned}$$

The differentials lie on different surfaces and, therefore, are independent vectors. The point of the space-time and the collection of differentials of different orders (support elements) form a space with larger dimension, namely the Finsler space (the space of support elements). The kinematic quantities are expressed in terms of the corresponding differentials:

$$\begin{aligned} u^0 &= \frac{dx^0}{d\tau} = c\frac{dt}{d\tau}, \qquad u^\alpha = \frac{dx^\alpha}{d\tau}, \\ \dot{u}^\alpha &= \frac{d^2 x^\alpha}{d\tau^2}, \qquad \ddot{u}^\alpha = \frac{d^3 x^\alpha}{d\tau^3}, \ldots, \qquad \alpha = 1,2,3. \end{aligned} \tag{3.2}$$

Here, τ is the intrinsic time of particles, which is invariant relative to changes of the reference system and the laboratory time t. If the reference system is changed in the space-time, the values of coordinates are connected by the relations with nonzero Jacobian:

$$x^{\alpha'} = \varphi^{\alpha'} \left(x^0, x^1, x^2, x^3 \right), \qquad \det \left| \frac{\partial x^{\alpha'}}{\partial x^\beta} \right| \neq 0. \tag{3.3}$$

Seven degrees of freedom in the Finsler space are physically obvious: they are four coordinates of the four-dimensional space-time, x^0, x^1, x^2, x^3, and three velocities in the coordinate space, $u^1 = \dfrac{dx^1}{d\tau}$, $u^2 = \dfrac{dx^2}{d\tau}$, $u^3 = \dfrac{dx^3}{d\tau}$. The definition of 4-dimensional velocity includes a new element, the intrinsic time.

Moreover, the eighth degree of freedom with dimension of velocity, $u^0 = c\dfrac{dt}{d\tau}$, appears.

Since the eighth degree of freedom u^0 is directly related to the flow of the physical time inside the ensemble of interacting particles, it is natural to assume the connection of this eighth degree of freedom with the physical properties of irreversibility of the system, degree of its "openness," and flows of the entropy in the system of particles. This point will be clarified in the subsequent study of the connection between the geometry and the processes of evolution.

The curvature of the stratified space-time is defined by the coherent acceleration of the system (and, hence, by mass forces). The category of motion of particles is included in the space of support elements on the same primary level as the category of space-time. Moreover, the forces are considered as a factor forming the properties of the space-time and the possible motions, which are already connected continuously with the image of a particle.

The ordinary phase space differs from the space of support elements in the following. The velocities in the phase space occupy the whole region in a vicinity of the corresponding point of the coordinate space, whereas the velocities in the space of support elements are in the plane tangent to the world lines passing through the given point of the space-time and are obtained by the differentiation along the world lines of particles.

This results in that the velocities in the Finsler space of support elements, as distinct from the Riemann space, are transformed always by a linear law, even for the arbitrary nonlinear transformations of coordinates (3.3).

In other words, the 8-dimensional Finsler space with coordinates $\left(x^0, x^1, x^2, x^3, u^0, u^1, u^2, u^3\right)$ is characterized by the transformations

$$x^{\alpha'} = \varphi^{\alpha'}\left(x^0, x^1, x^2, x^3\right), \qquad u^{\alpha'} = u^{\alpha}\frac{\partial x^{\alpha'}}{\partial x^{\alpha}}. \tag{3.4}$$

Elements $a^{\alpha} = a^{\alpha}\left(x^{\sigma}, u^{\sigma}\right)$ form a contravariant vector, if they are transformed as the vector of velocities u^{α}(see (3.4))

$$a^{\alpha'} = a^{\alpha}\frac{\partial x^{\alpha'}}{\partial x^{\alpha}}, \tag{3.5}$$

and $a_{\beta} = a_{\beta}\left(x^{\sigma}, u^{\sigma}\right)$ form a covariant vector, if they are transformed with the help of the relations

$$a_{\beta'} = a_{\beta}\frac{\partial x^{\beta}}{\partial x^{\beta'}}. \tag{3.6}$$

According to the Hausdorff theorem of the metrics of topological spaces, the general metric of the space $\tilde{d}s^2 = ds^2 + ds_u^2$ can be set by a sum of the independent metrics of the space-time ds^2 and the metric of the tangent bundle, i.e., that of the space of velocities $ds_u^2 = q_{ik}du^idu^k$.

In this case, the metric of the space-time part can be of two basically different types:

1) metric tensor of the space-time depends only on coordinates and the time: $g_{ik} = g_{ik}\left(x^l\right)$;
2) metric tensor of the space-time depends on velocities (and, possibly, accelerations) implicitly, as on parameters: $g_{ik} = g_{ik}\left(x^l, \left\{\vec{u}, \dot{\vec{u}}, \ddot{\vec{u}}, \dddot{\vec{u}}, \ldots\right\}\right)$.

In the first (isotropic) case corresponding to a weak deviation of the collective system from the equilibrium without rearrangement of the internal structure, the metric depends only on coordinates.

In the second case, we have an anisotropic scenario corresponding to the collective system with self-organizing internal structure. The dependence of the metric coefficients on velocities (and/or accelerations) leads to a specific anisotropy of the space-time: *in any infinitely small region, the space-time is anisotropic, and its properties depend on the direction of motion and the acceleration of particles.*

Namely the anisotropy of the space-time, which arises obviously at the coherent acceleration of the system, is the main reason for the formation of macroscopic quantum (coherent, nonlocal) objects of the shell type.

The particles of a shell, which are organized in a collective state, i.e., MQO, form a noninertial reference system. The acceleration of this collective reference system is reflected in the space-time curvature resulting in the difference between the intrinsic and laboratory times, which can cause, as will be shown below, the explosive change of space-time scales.

3.2 Geodesic lines in an evolving system

Under the action of mass forces on a system of particles, the particles are identically accelerated and form a noninertial system. In a vicinity of the arbitrary point of the space-time, the dynamics of the system is set by covariant accelerations and, hence, by covariant derivatives of the velocity. In turn, the covariant derivatives of the velocity are given by the tensor of accelerations. As follows from the Gauss principle and the principle of dynamical harmonization, the constraints in the system are changed with the help of a variation of accelerations. Therefore, the constraints between kinematic quantities and the limitations imposed on them are determined by coherent accelerations. It is clear that, in this case, one of the significant quantities is the covariant velocity differential $D_\beta u^\alpha$, whose value is set by the tensor of accelerations $a^\alpha_\beta\left(x^\sigma, u^\sigma\right)$ and is determined by the structure of constraints:

$$D_\beta u^\alpha = \frac{\partial u^\alpha}{\partial x^\beta} + C^\alpha_{\beta\gamma} u^\gamma = a^\alpha_\beta\left(x^\sigma, u^\sigma\right). \tag{3.7}$$

Here, $C^\alpha_{\beta\gamma}$ are the generalized coefficients of connectedness, which coincide with the Chistoffel symbols $\Gamma^\alpha_{\beta\gamma} = \frac{1}{2} g^{\alpha\sigma}\left(\frac{\partial g_{\beta\sigma}}{\partial x^\gamma} + \frac{\partial g_{\sigma\gamma}}{\partial x^\beta} - \frac{\partial g_{\beta\gamma}}{\partial x^\sigma}\right)$ in the simplest case where $a^\alpha_\beta\left(x^\sigma, u^\sigma\right) = 0$ and can be expressed in terms of the space-time metric. Let us now calculate the covariant acceleration by using the covariant velocity differential:

$$\frac{Du^\alpha}{d\tau} = \frac{\left(D_\beta u^\alpha\right) dx^\beta}{d\tau} = a^\alpha_\beta \frac{dx^\beta}{d\tau}. \tag{3.8}$$

If $a^\alpha_\beta \frac{dx^\beta}{d\tau} \neq 0$, then there exists a nonzero external covariant acceleration, and the system undergoes the action of external forces. But the situation where $a^\alpha_\beta \frac{dx^\beta}{d\tau} = 0$ can be also realized; i.e., the covariant acceleration of the system is zero, and no external forces affect the system. However, the coherent acceleration can be present, nevertheless, inside the system, the constraints, $a^{\alpha}_{\beta}(x^{\sigma}, u^{\sigma}) \neq 0$, can hold, and the influence of the motion of particles on the space-time metric can be revealed in a change of both spatial and temporal scales.

Let us set the tensor of accelerations in the simplest form:

$$a^{\alpha}_{\beta} = a\left(\delta^{\alpha}_{\beta} - \frac{u^{\alpha}u_{\beta}}{c^2}\right), \qquad u^{\alpha}u_{\alpha} = c^2, \qquad a = const. \tag{3.9}$$

In this case, the force action on the system is absent. Indeed, for tensor (3.9), we have

$$\begin{aligned} a^{\alpha}_{\beta}\frac{dx^{\beta}}{d\tau} &= a\left(\delta^{\alpha}_{\beta} - \frac{u^{\alpha}u_{\beta}}{c^2}\right)\frac{dx^{\beta}}{d\tau} = a\left(\delta^{\alpha}_{\beta} - \frac{u^{\alpha}u_{\beta}}{c^2}\right)u^{\beta} \\ &= a\left(u^{\alpha} - \frac{u^{\alpha}u_{\beta}u^{\beta}}{c^2}\right) = 0. \end{aligned} \tag{3.10}$$

Relations (3.7) are 16 equations for four unknowns u^{α}; i.e., these equations have no solutions in the general case with arbitrary generalized coefficients of connectedness $C^{\alpha}_{\beta\gamma}$. The condition of existence of solutions of Eqs. (3.7) imposes certain limitations on the quantities $C^{\alpha}_{\beta\gamma}$ or $\Gamma^{\alpha}_{\beta\gamma}$ and, hence, on the metric.

The dynamics of particles occurs along the geodesic lines in the Finsler space, whose geometry varies in the general case in correspondence with the running evolution of the internal structure of particles and the system on the whole according to the equations

$$\frac{d^2x^i}{dt^2} + C^i_{jk}\frac{dx^j}{dt}\frac{dx^k}{dt} + F^i = 0. \tag{3.11}$$

E. Cartan and J. Schouten showed that, by the differential equations (3.11), it is possible to restore the geometry, i.e., the metric. Conditions (3.8) are consistent with a nonstationary metric:

$$\begin{aligned} ds^2 = \left(dx^0\right)^2 - \sigma^2\left(x^0\right) g_{\alpha\beta}\left(x^1, x^2, x^3\right) dx^{\alpha}dx^{\beta}, \qquad \alpha, \beta = 1, 2, 3, \\ \sigma\left(x^0\right) = \exp_q\left(\frac{x^0}{c\tau_{eff}}\right), \end{aligned} \tag{3.12}$$

where $g_{\alpha\beta}\left(x^1, x^2, x^3\right)$ is the spatial part of the Riemann metric. In this metric,

$$\kappa_{10,10} = \frac{a^2_{cog}}{c^4}\exp_q\left(\frac{x^0}{c\tau_{eff}}\right), \qquad \kappa = 2\frac{a^2_{cog}}{c^4} \tag{3.13}$$

are the main independent component of the curvature tensor and the scalar curvature of the space-time, which depend on the parameter q closely connected with the order parameter and the coefficient of impactness of a driver.

As was shown in Section 2, the controlled change of the entropy of the system can be an efficient driver. In other words, it can induce a coherent acceleration of the system and, hence, the space-time curvature. The strongest changes of the entropy initiate the explosive processes of growth or decay of structures. Let us consider an important example of the explosive clusterization in the system of monomers (see [1,5]) and calculate the entropic forces and the space-time curvature created by the process of clusterization.

The system of particles aggregating as a result of binary contacts is a set of clusters of various sizes. The distribution of clusters over sizes, i.e., the concentration of clusters with size k (clusters consisting of k monomers) as a function of the time is described by the system of reactions $A_{k0} + A_{k0} \to A_{2k0}$, $A_{k0} + A_{2k0} \to A_{3k0} \dots$.

In this case, the equation for the concentrations C_k of clusters including k monomers can be written in the form of the Smoluchowski coagulation equation [41]. This equation involves the competition of two processes: the sticking of a cluster with monomers, i.e., the increase of the size of a cluster, and the breaking of clusters, i.e., the growth of the number of clusters with low masses. For the probability K_{ij} of the sticking of clusters with sizes i and j, we take the approximation such that this probability is proportional to the product of the surface areas of the input clusters: $K_{ij} \propto (ij)^{2/3}$. According to the solution of the Smoluchowski equation, the time dependence of the mean size $s(t)$ of a cluster manifests the explosive behavior:

$$s(t) = \frac{s_0}{\left(1 - \dfrac{t}{t_c}\right)^{1/6}}. \tag{3.14}$$

Here, t_c is the time moment of the phase transition into the state of a global cluster. By the order of magnitude, this time is equal to several collision times. The explosive growth (3.14) of a cluster is directly related to the change of the number of constraints in the system and, hence, to the change of the entropy. The particles, which are organized into the collective state (i.e., MQO), form the own reference system. The acceleration of this collective reference system arosen due to the action of a mass force is reflected in the space-time curvature and, in particular, in the difference between the intrinsic and laboratory times. In this case, the local time and its intervals differ from the corresponding values in the laboratory reference system in agreement with metric (3.12):

$$\tau/t_0 = \ln\left(1 + (1-q)\left(\frac{x^0}{c\,t_0}\right)\right)^{\dfrac{1}{1-q}} = \ln\left(\exp_q\left(\frac{x^0}{c\,t_0}\right)\right). \tag{3.15}$$

The parameter q is connected with the order parameter η with the help of relations obtained in [1]. It follows from formula (3.15) that the laboratory and intrinsic times coincide for the nonequilibrium parameter equal to 1. As the degree of nonequilibrium and the deviation of the parameter q from 1 increase, the intrinsic time rapidly decelerates or accelerates as compared with the laboratory one, by depending on the sign of the acceleration of the nonequilibrium reference system (NRS) (and, respectively, on the direction of the deviation of q from 1).

Dependence (3.15) of the ratio of the intrinsic time to the laboratory one on the laboratory time for various values of the parameter of nonequilibrium q is presented in Fig. 3.1.

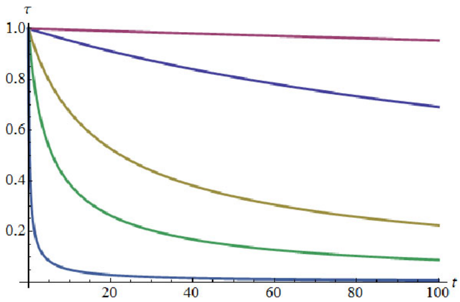

Fig. 3.1. Ratio of the intrinsic time to the laboratory one versus the laboratory time for values of the parameter $q = 1.001,\ 1.01,\ 1.1,\ 1.30,\ 1.90$. Curves correspond to values increasing from top to bottom. For q=1, the laboratory and intrinsic times coincide, and their ratio is equal to 1.

The variation of the intrinsic time as compared with the laboratory one leads, in turn, to a decrease of the light velocity in the region occupied by the growing cluster. The decrease of the light velocity $c = \dfrac{c_0}{\sqrt{n}}$ manifests itself as the effect of increasing the refractive index

$$n = \frac{1}{\left(\ln\left(\exp_q\left(\dfrac{x^0}{c\,t_0}\right)\right)\right)^2}$$

near the growing cluster. It is worth noting that the described effect of increasing the refractive index and decreasing the light velocity in a vicinity of growing crystals was discovered, in fact, experimentally as earlier as 1902 and was described in work [25].

We note that the geodesic world lines of particles, along which the particles move in correspondence with the principle of dynamical harmonization, are, in fact, the characteristics of a kinetic equation of the Vlasov type. This allows us to write the very kinetic equations and to realize the connection between the dynamical and statistical descriptions of the evolution of systems with varying constraints. Below, we will analyze the solutions of covariant kinetic equations, which allow one to answer the majority of questions posed by the "shell" model of self-organization (see [1]) and to obtain the equations of dynamical harmonization for it.

3.3 Covariant kinetic equations for particles and their solutions

In order to describe the many-particle interactions, the mean accelerations of particles $\langle \dot{\vec{u}} \rangle = \dfrac{1}{m}\vec{F}_B$ in the Vlasov equation (2.36) are usually determined by self-consistent electromagnetic fields. As follows from Section 2, the analysis of the evolution of MQO should consider not only the fields of fundamental interactions, but also the entropic forces, which modify, in fact, the Vlasov equation, supplementing it by the collision integral in the divergent form:

$$\begin{gathered}\frac{\partial f\left(\vec{r},\vec{u},t\right)}{\partial t}+div_{\vec{r}}\left(\vec{u}\,f\left(\vec{r},\vec{u},t\right)\right)+div_{\vec{u}}\left(\langle\dot{\vec{u}}\rangle f\left(\vec{r},\vec{u},t\right)\right)=div_{\vec{u}}\left(\vec{j}_S\right);\\ \vec{j}_S=\left(-\frac{1}{m}\nabla_r\left(\omega_{eff}S_q\right)\right)f\left(\vec{r},\vec{u},t\right);\\ \langle\dot{\vec{u}}\rangle=\frac{1}{m}\vec{F}_B,\;\omega_{eff}\approx\frac{2\pi}{\tau_{eff}}.\end{gathered}\tag{3.16}$$

The action of a mass entropic force on the system of particles and the continuously related acceleration compel the system to reconstruct its internal structure and, thus, to evolve in a tangent bundle of the space-time according to the variational principle of dynamical harmonization.

A change of the structure causes variations of the distribution functions of particles of the system, which are related to the processes of localization or delocalization and, in turn, affect the dynamics of particles through the entropic forces. The interrelation of the evolution of a system in the corresponding layer and the four-dimensional basis of the Finsler space-time is realized through the accelerations and the distribution functions.

The kinetic equation (3.16) for the particles composing MQO can be represented in the 8-dimensional space of support elements in the covariant form

$$\widehat{D}iv_r\left(\vec{u}\,f\right)+div_u\left(\left\langle\frac{\widehat{D}\vec{u}}{d\tau}\right\rangle f\right)=0.\tag{3.17}$$

Since

$$\begin{gathered}\widehat{D}iv_r\left(\vec{u}\,f\right)=u^\alpha\widehat{D}_\alpha f+f\widehat{D}_\alpha u^\alpha,\\ div_u\left(\left\langle\frac{\widehat{D}\vec{u}}{d\tau}\right\rangle f\right)=\frac{\partial}{\partial u^\alpha}\left(\left\langle\frac{\widehat{D}\vec{u}}{d\tau}\right\rangle^\alpha f\right),\\ \widehat{D}_\alpha f=\frac{\partial f}{\partial x^\alpha}-\Gamma^\sigma_{\alpha\gamma}u^\gamma\frac{\partial f}{\partial u^\sigma},\\ \Gamma^\sigma_{\alpha\gamma}=\frac{1}{2}g^{\mu\sigma}\left(\frac{\partial g_{\mu\alpha}}{\partial x^\gamma}+\frac{\partial g_{\mu\gamma}}{\partial x^\alpha}-\frac{\partial g_{\alpha\gamma}}{\partial x^\mu}\right),\end{gathered}$$

we can write (3.53) by components:

$$u^\alpha\frac{\partial f}{\partial x^\alpha}-\Gamma^\sigma_{\alpha\gamma}u^\gamma\frac{\partial f}{\partial u^\sigma}+\frac{\partial}{\partial u^\alpha}\left(\left\langle\frac{\widehat{D}\vec{u}}{d\tau}\right\rangle^\alpha f+P_s\right)=0.\tag{3.18}$$

Let us analyze the solutions of the kinetic equation in a significant partial case where the explicit contribution of the divergence $div_{\vec{u}}\,(.)$ to the kinetic quations can be neglected. Then the covariant equation of quasistationary states takes the form

$$u^\alpha\frac{\partial f}{\partial x^\alpha}-\Gamma^\sigma_{\alpha\gamma}u^\gamma\frac{\partial f}{\partial u^\sigma}=0.\tag{3.19}$$

This approximation is valid in two cases:

- if the external forces are completely absent, and the flow in the phase space, $P_S = 0$ (this case corresponds to the full equilibrium of the system);
- if the external forces do not act directly in the system, but the flows of energy, particles, or the entropy are constant in the phase space, $P_S = const$ (this case corresponds to a strong deviation from the equilibrium).

In the nonequilibrium case, the kinetic equations for systems with varying constraints have quasipower and power solutions [39–40]. In this case, the exponent of a solution depends on the flows created in the system [40], the external forces inside the dynamical system of particles can be neglected, and the whole action of mass forces is determined by the entropy flows that are generated on the boundary of an MQO-shell and disappear in the orthogonal direction, where the delocalization and the growth of a structure in the system occur.

Consider the solutions of the kinetic equation (3.19), which are isotropic in the space of velocities and stationary in the laboratory time, i.e., we assume that

$$\partial f/\partial x^0 = 0; \qquad \partial g_{\alpha\beta}/\partial x^0 = 0, \qquad g_{0i} = 0, \quad i = 1, 2, 3.$$

We emphasize that this stationary state does not assume the independence of the intrinsic time. In the indicated approximation, we have

$$\Gamma^0_{00} = \Gamma^0_{ik} = 0; \qquad \Gamma^k_{i0} = 0; \qquad \Gamma^0_{0i} = \frac{1}{2g_{00}} \frac{\partial g_{00}}{\partial x^i}; \qquad \Gamma^i_{00} = -\frac{1}{2} g_{ik} \frac{\partial g_{00}}{\partial x^k}.$$

Separating the variables, we present the distribution function in the form

$$f\left(x^\alpha, u^\alpha, u^0, t\right) = f\left(x^\alpha, u^\alpha, u^0\right) = \rho\left(x^\alpha\right) \psi\left(u^2\right) \psi_0\left(u^0\right), \tag{3.20}$$

$$(1-q)\,\xi^2 = g_{\alpha\beta}\xi^\alpha\xi^\beta, \qquad \xi^\alpha = \frac{u^\alpha}{\sqrt{-(1-q)\,g_{oo}u^0}}, \qquad u^2 = u_\alpha u^\alpha. \tag{3.21}$$

Substituting (3.21) in (3.20) and (3.19) and separating the variables, we obtain the system of equations, which is exactly solvable:

$$\rho\left(x^i\right) = \rho_0 \exp_q\left(-\frac{U(x)}{w(q)}\right), \qquad \psi_0\left(u^0\right) = \psi_0(0) \frac{1}{u_0^{\frac{q_{cr}}{q_{cr}-1}}},$$
$$\psi\left(\xi^2\right) = \left(\exp_q\left(-\xi^2\right)\right)^q. \tag{3.22}$$

The obtained solutions reflect the fact that, in the absence of the entropy flow (i.e., for $q = 1$), the homogeneous equilibrium case is realized, since the distribution function $\psi\left(\xi^2\right)$ over velocities (or energies) transits in the Maxwell distribution function, the distribution function $\rho\left(x^i\right)$ becomes the Boltzmann distribution, and the function $\psi_0\left(u^0\right)$ is constant, so that there is no difference between the local and laboratory times. If the entropy flows are present in the system ($q \neq 1$), the exponential distribution functions over energies and coordinates become the quasipower ones.

3.4 Anisotropy of states in a noninertial dynamical system

The action of the entropic fields of mass forces initiating the coherent acceleration of particles transforms the whole system into a noninertial reference system (NRS). The inertial reference systems (IRS) are associated with the absence of flows and coherent accelerations in the reference system, i.e., with equilibrium systems without evolution. *On the contrary, namely the action of mass forces on IRS and its transformation in NRS with some coherent acceleration and flows in it compel the particles to the evolution, i.e., to a change of the internal structure of constraints between particles, energy of constraints, and mass defect of the system.*

In the general case, the system (like a spheroid), which is isotropic at the initial time moment and has a spatial distribution of particles with characteristic scale l_0 in the inertial system, evolves in the noninertial system in a deformed anisotropic state with the number of external space scales more than 1 and becomes similar to an ellipsoid-"pancake". In a sufficiently general case, we may distinguish two basically different orthogonal directions:

- along the direction of the acceleration;
- in the plane orthogonal to the acceleration.

By controlling the anisotropy of the wave function of a quantum system, it is possible to control the localization of the system and, hence, the probabilities of the processes of evolution related to the overcoming of energy barriers. This is obvious even at the analysis of pure states.

The Heisenberg uncertainty relation for one degree of freedom takes the form $\Delta x \Delta p_x \geqslant \frac{\hbar}{2}$. In the complete phase space (e.g., for three degrees of freedom), this relation determines the size of its minimal cell:

$$\Delta\Omega_{ph} \geqslant \frac{\hbar^3}{8}. \tag{3.23}$$

In the simplest case, we will take the anisotropy of the evolution into account, by introducing two scales as macroscopic geometric characteristics of the system instead of its single radius:

- $l_- < l_0$, in the direction of the "flattening" of the system;
- $l_+ > l_0$, in the orthogonal directions, along which the "flattening" happens.

The less scale l_- can be called a scale of spatial coherence of the system, which characterizes the "pancake" thickness. The larger scale l_+ is a characteristic scale of the interaction, which characterizes the maximal length of correlations in the system.

Consider the case of a strong deviation from the local equilibrium. Though the external forces do not act directly in the system, the entropy flow is constant in the phase space, and the density distribution in the bounded system and the erosion of the boundary happen in agreement with a distribution function of the type (3.22), rather than with equilibrium distribution functions.

The density distribution in the system with mass number A that consists of monomers with mass number and characteristic size l_{str} is described at a distance Δr from the center by the squares of the corresponding wave functions with characteristic scale $l_0(\eta) = l_{str}\left(\dfrac{A}{A_{str}}\right)^{\dfrac{\eta+1}{3+\eta}}$:

- $\exp\left(-\left(\dfrac{\Delta r}{l_0}\right)^2\right)$, for the equilibrium noncoherent part of the system;
- $\exp_q\left(-\left(\dfrac{\Delta r}{l_0(q)}\right)^2\right)$, for the coherent part of the system with regard for the action of entropic forces.

Without the entropy flow (for $q = 1$), a homogeneous equilibrium state is realized. In this case, the distribution function over velocities (or energies) transits in the Maxwell distribution. If the entropy flows are present in the system ($q \neq 1$), the exponential distribution functions over energies and coordinates becomes quasipower one.

The processes of self-organization essentially depend on the direction of the entropy flow, since there exist two types of behavior of the distribution function: localization and delocalization. They correspond to different characters of the behavior of the function $\exp_q\left(-\left(\dfrac{\Delta r}{l_0(q)}\right)^2\right)$ for $q \leqslant 1$ and $q > 1$. The parameter q is determined from (2.41)–(2.42) and is connected with the order parameter $0 \leqslant \eta \leqslant 1$ by the relations

$$q(\eta) = \begin{cases} q_- = 1-\eta, & q \leqslant 1 \\ q_+ = \dfrac{1}{1-\eta}, & q > 1 \end{cases}. \tag{3.24}$$

As the order parameter increases, the character of decay of the wave functions in the space passes from exponential to power (see 3.22). The dependences of the scales on the order parameter in the directions of delocalization and localization take the form

$$l_+ \approx l_0 \frac{1}{\left(1-\dfrac{\eta}{\eta_{\max}}\right)^{\gamma_{str}}}, \qquad l_- \approx l_0\left(1-\frac{\eta}{\eta_{\max}}\right), \\ \gamma_{str} \approx 1.83, \qquad \eta_{\max} = 0.5. \tag{3.25}$$

The phase volume is a product of volumes in the coordinate and momentum spaces, and the volume in the coordinate space is a product of volumes in the coherent direction $\Delta\Omega_-$ and in the direction $\Delta\Omega_+$ orthogonal to it. Thus, we have $\Delta\Omega_{ph} = (\Delta\Omega_+\Delta\Omega_-)\,\Delta\Omega_p$.

The volume in the plane orthogonal to the direction l_- can be estimated as $\Delta\Omega_+ \approx \pi l_+^2$. We consider that the minimal size of a cell l_- is attained in the coherent state: $(l_-)_{\min} \approx \dfrac{\hbar}{2p_f}$. Since the volume in the momentum space $\Delta\Omega_p \approx \dfrac{4}{3}\pi p_f^3$, we have

$$l_+ \geqslant \frac{1}{2\pi}\left(\frac{\hbar}{p_f}\right)^{3/2}\sqrt{\frac{3}{8\,l_-}}. \tag{3.26}$$

These relations are valid, if there are no correlations between coordinates and momenta. However, if the system of particles turns out in the state with coherent acceleration, then, as is seen from the above discussion, the space-time metric is changed. Hence, the geodesic lines and the dynamics of particles are changed as well. It is clear that, in this case, due to the anisotropic space-time curvature (and to the coherent acceleration), the strong correlations appear:

$$r_{xp} \approx \eta^{2(\gamma_{str}-1)/3}. \tag{3.27}$$

In the general form, the correlations r_{xp} between momenta and coordinates were considered by Robertson and Schrödinger [42], who wrote the uncertainty relation in the form

$$\Delta x \Delta p_x \geqslant \frac{\hbar}{2\sqrt{1-r_{xp}^2}}. \tag{3.28}$$

Let us return to relation (3.23). With regard for the correlations r_{xp}, we can transform it into the form similar to (3.26):

$$l_+ \geqslant l_- \frac{1}{2\pi}\sqrt{\frac{3}{8}}\left(\frac{\hbar}{p_f l_-}\frac{1}{\sqrt{1-r_{xp}^2\left(a_{cog}\right)}}\right)^{3/2} \tag{3.29}$$

As was shown in [1], this relation at the coherent acceleration of the system of particles yields the explosive delocalization of a state of the system in the direction orthogonal to the direction of the acceleration. As the shell thickness decreases, the energy becomes quantized in the direction perpendicular to the shell surface. In other words, the dispersion of momenta of particles around the momenta localizing themselves is decreased. In this case, the dispersion of coordinates of particles along the surface is sharply increased.

It was shown in [43] that, on the basis of estimates (3.27), it is possible to develop a model of the overcoming of barriers by an oscillator located in a nonstationary external field ensuring the growth of correlations.

3.5 Equations of dynamical harmonization of a system with varying constraints and the geometry of a stratified space-time

The most general representation of the laws of dynamics and evolution of the systems of particles is given by the variational principle of dynamical harmonization [1], which is a generalization of the Gauss and Hertz principles for the systems with varying internal structure and binding energy. It assumes that the self-organization of a system occurs as a result of the variation of the structure of constraints between its particles (elements) as a response to their coherent acceleration.

By the Gauss principle, those positions that will be occupied by the points of the system at the time moment $t+\tau$ in their real motion are distinguished between all positions admissible by constraints by the minimal value of compulsion measure $Z_G = \sum_{i=1}^{N} m_i s_i^2$ (here, $s_i\left(\delta a\right)$ is the length of a vector between the points representing the true and any possible positions of a point; it depends only the acceleration variation δa).

The optimal variations of accelerations, as was shown by Hertz, correspond to the minimal curvature of the trajectories of particles. This means that the dynamics of particles is realized along the geodesic lines corresponding to definite constraints. The notion of motion includes also a rearrangement of both the structure of the system and the field of constraints of its structural elements. While the system moves, its fractal dimension D_f and binding energy $B(D_f)$ [1], which are determined by the packing of monomers composing the system, are changed.

Changes of the structure and constraints in the system vary obviously the masses of the system and its components (i.e., the inertia or sensitivity of the system relative to the external forces acting on it).

As was shown in [1], the evolution of an internal structure of the system is determined by the principle of dynamical harmonization, which involves the possibility of a change of constraints in the system: *under the action of external, F_i, and mass, F_{str}, forces, the system varies its trajectory and the structure in order to be consistent with the external medium and external actions, by minimizing the generalized compulsion function,*

$$Z_{dh} = \sum_{i=1}^{N} \left(m_i\left(D_f\right) w_i - \left(F_i + \left(F_{str}\right)_i\right)\right)^2,$$
$$m_i\left(D_f\right) = \left(m_{i0} - \delta m_i\left(D_f\right)\right), \qquad \delta m_i\left(D_f\right) = \frac{B_i\left(D_f\right)}{c^2}, \tag{3.30}$$

with regard for the variations of all constraints in the system (respectively, with regard for the variation of the binding energy $B_i(D_f)$).

In other words, the system tends to make the trajectories of its compulsory motion under the action of mass forces to be maximally close to the trajectories of the own nonperturbed motion.

Since a change of the internal structure of the system is regularly related to a change of its mass $m_i(D_f)$, the processes accompanied by a change of the structure are most efficient at the evolution of the system, because they can serve as both a source of energy and a means of its accumulation for the very evolution. *It is obvious that the control over a system on the basis of the laws of evolution of its constraints (the principle of dynamical harmonization of the systems with varying constraints) is the unique efficient way o the realization of desired transformations in the system due to the use of its internal energy resources, rather than due to the direct "violence" with the use of only the external energy.*

The tool to initiate the processes of self-organization of a structure of constraints in the system is a general dominating perturbation specially selected for the given system and the appropriate coherent acceleration of the ensemble of particles composing the system.

Because a change of the structure is continuously connected with changes of the entropy and the information, the principle of dynamical harmonization describes simultaneously the targeted exchange by information and the entropy between the system and the environment. This means that the space-time geometry (curvature) and the evolution of an internal structure of the system of particles are indivisible. Such a situation is a natural continuation of properties arising in the dynamical systems with varied constraints between elements of the system under the optimization of their control.

In this case,

- the state of the system is set by a vector in the configurational space of a dynamical system,
- the constraints are set by a matrix of the constraint coefficients,
- the control is realized by the external vector of control, being the vector of forces acting on the appropriate components of the system.

An analogous situation arises also at the evolution of the system represented by a collection of monomers:

- the state of such a system is set by the positions of particles in the four-dimensional space-time and their velocities, which are tangent to the trajectories of particles at the given point and, hence, belong to a tangent bundle of the space-time;
- the constraints between monomers are characterized by their energies depending on the structure of the system, which is characterized, in turn, by the dimension (the dimension of a structure of constraints in the system) and the entropy (information);
- the evolution of the system occurs in the tangent bundle of the space-time and is governed by the equations of dynamical harmonization in a non-inertial reference system with given coherent acceleration. The evolution forms the entropic forces that define the dynamics of the system of particles with varying constraints in the space-time. The coordinates in layers are the accelerations of all orders; additionally, we have a layer with the fractal dimension of a system of constraints (or with their entropy) as a coordinate;
- the dynamics of the system of particles occurs in the anisotropic space-time with a curvature that is determined by the acceleration of the non-inertial reference system depending on the entropic forces;
- the control is realized by the external vector of control, which sets the contributions to the appropriate components of coherent accelerations of the system.

As was shown in the previous section, the action of mass forces on the system causes the rapid (as compared with a quasiequilibrium case) "flattening" of the distribution function, which corresponds to the presence of the negative flows of entropy in the system (or, what is the same, the flow of information in the system), ensures an increase of the volume, and, by this, modifies the dynamics of scales. In the general case, as the order parameter increases and the fractal dimension varies, let the localization scale l_- be decreased, and let the scale l_+ be increased as compared with the equilibrium values by the relations

$$l_- = g_- \left(D_f, \delta\right); \qquad l_+ = g_+ \left(D_f, \delta\right). \tag{3.31}$$

The estimation of these functions was mase above (see (3.25)). In agreement with the principle of dynamical harmonization, the equations of evolution are determined by a minimum of the dynamical harmonization functional Z_{dh} at the variation of the accelerations of the scales of localization and delocalization of the system (respectively, w_- and w_+):

$$\begin{gathered} Z_{dh} = \frac{1}{2}(m\, w_+ - F_+)^2 + \frac{1}{2}(m\ w_- - F_-)^2, \\ m = m_0 - B_A\left(\eta, \delta\right)/c^2. \end{gathered} \tag{3.32}$$

The variation by Gauss assumes the tangent plane to a current point on the trajectory of a particle to be fixed, and the transition from the dynamics in the coordinate space to that in the Finsler space is very simple. Then the variations by Gauss look as those in a tangent plane with second-order tangency at the fixed plane with first-order tangency. *The variations of accelerations (i.e., of vectors in the corresponding different planes) of all orders are independent. Therefore, the variations by Gauss lead to that the relations for the variations of accelerations are similar to those for the variations of the corresponding coordinates. Hence, the below-presented relations for accelerations do not incliude the first derivatives of the constraint equations*:

$$\begin{aligned} w_1 &= \frac{d^2}{dt^2} l_1 = \gamma_{11}\ddot{D}_f + \gamma_{12}\ddot{\delta}, \qquad \gamma_{11} = \frac{\partial^2 g_1}{\partial^2 D_f}, \quad \gamma_{12} = \frac{\partial^2 g_1}{\partial^2 \delta}; \\ w_2 &= \frac{d^2}{dt^2} l_2 = \gamma_{21}\ddot{D}_f + \gamma_{22}\ddot{\delta}, \qquad \gamma_{21} = \frac{\partial^2 g_2}{\partial {D_f}^2}, \quad \gamma_{22} = \frac{\partial^2 g_2}{\partial^2 \delta}. \end{aligned} \tag{3.33}$$

Here,

index 1 corresponds to the direction of delocalization x_+ and the scale of delocalization l_+,

index 2 corresponds to the direction of localization x_- and the scale of localization l_-.

Substituting the formulas for the accelerations in Z_{dh}, we obtain the dynamical harmonization functional depending on the accelerations of the fractal dimension and the deformations of scales:

$$\begin{aligned} Z_{dh}\left(\ddot{D}_f, \ddot{\delta}\right) =& \frac{1}{2}\left(\gamma_{11}\ddot{D}_f + \gamma_{12}\ddot{\delta} - \frac{F_1(D_f,\delta)}{m(D_f,\delta)}\right)^2 \\ &+ \frac{1}{2}\left(\gamma_{21}\ddot{D}_f + \gamma_{22}\ddot{\delta} - \frac{F_2(D_f,\delta)}{m(D_f,\delta)}\right)^2. \end{aligned} \tag{3.34}$$

The condition of minimum of the dynamical harmonization functional with respect to the accelerations of the fractal dimension and the deformations of scales,

$$\frac{\partial Z_{dg}(\ddot{D}_f,\ddot{\delta})}{\partial \ddot{D}_f} = 0, \qquad \frac{\partial Z_{dg}(\ddot{D}_f,\ddot{\delta})}{\partial \ddot{\delta}} = 0,$$

leads to the system of differential equations determining the evolution of a dymanical system with varying constraints:

$$\ddot{D}_f = \frac{a_{22}G_1 - a_{12}G_2}{a_{11}a_{22} - a_{12}a_{21}}; \qquad \ddot{\delta} = \frac{-a_{21}G_1 + a_{11}G_2}{a_{11}a_{22} - a_{12}a_{21}}, \tag{3.35}$$

$$\begin{aligned} a_{11} &= \left((\gamma_{11})^2 + (\gamma_{21})^2\right), \qquad a_{12} = (\gamma_{11}\gamma_{12} + \gamma_{21}\gamma_{22}), \\ G_1 &= \gamma_{11}\frac{F_1}{m} + \gamma_{21}\frac{F_2}{m}, \\ a_{21} &= (\gamma_{11}\gamma_{12} + \gamma_{21}\gamma_{22}), \qquad a_{22} = \left((\gamma_{12})^2 + (\gamma_{22})^2\right), \\ G_2 &= \gamma_{12}\frac{F_1}{m} + \gamma_{22}\frac{F_2}{m}. \end{aligned} \tag{3.36}$$

The obtained equations for the order parameter and a deformation of the probability density distribution of the system and the very variational principle of dynamical harmonization, from which the equations are deduced, are the basis of the theory of self-organization of systems with varying constraints.

In the simple situation where the deformation varies much more slowly that the internal structure, we may consider the deformation to be given. The equation describing the evolution of a structure, which has always time to tune itself to a given deformation, was obtained in [1]. In this case, the order parameter and the fractal dimension evolve according to the equation of dynamical harmonization, which has form of the Lagrange equation describing a change of the structure of the system with the use of the corresponding Lagrange function L_{str}:

$$\frac{d}{dt}\left(\frac{\partial L_{str}}{\partial \dot{D}_f}\right) - \frac{\partial L_{str}}{\partial D_f} = 0,$$
$$L_{str} = m_{str}\left(D_f\right) R_0 \frac{\dot{D}_f^2}{2} + sB_A\left(Z, D_f\right) A - U_{str}\left(D_f\right). \tag{3.37}$$

Here, $sB_A\left(Z, D_f\right)$ is the specific binding energy of a cluster per nucleon, $m_{str}\left(D_f\right)$ is the structural inertia of the system, $u_{D_f} = \dot{D}_f$ is the rate of variation of the fractal dimension, and $p_{D_f} = \dfrac{\partial L_{str}}{\partial \dot{D}_f} = m_{str}\left(D_f\right) R_0 u_{D_f}$ is the momentum of the system corresponding to its structurization.

Analogously to the Hertz principle, the principle of dynamical harmonization can be represented as the requirement of a minimum of the functional, being the length of a world line of particles in the Finsler space-time. As a result, we obtain the following statement of the variational principle of evolution: *the evolution of a system with constraints occurs along geodesic lines in the Finsler space-time with the curvature tensor corresponding to the evolution of internal constraints of the system, which are harmonized as a response to the coherent acceleration caused by the action of mass entropic forces.*

4 Electrophysical aspects of the interactions of particles and radiation with vacuum

First, it is pertinent to present the citation from [44]: “A reasonable staring point at the consideration of the problem of many bodies would be the question about the number of bodies for the problem to be posed.. . . The persons interesting in the exact solutions can find the answer, by looking at the history. For the Newton mechanics in the 18-th century, the problem of three bodies was unsolvable. After the creation (about the year 1910) of general relativity theory and quantum electrodynamics (about the year 1930), the problems of two bodies and a single one became unsolvable as well. In the modern quantum field theory, we meet the unsolvable problem without bodies (vacuum). So that if we are interested in the exact solutions, the zero number of bodies is too much”.

We note that the system of particles in a longitudinal electromagnetic field can form a noninertial reference system under a coherent acceleration, if the coefficient of domination exceeds 1. In this case, the system becomes nonequilibrium and open, in particular, for the interaction with vacuum.

The gravitational force acting between all bodies is the most known mass force. The modern theories of gravitation are based on general relativity theory developed by Einstein in 1915 [45]. The Einstein theory of gravitation is founded on the following assertions:

- The density and the pressure of a substance make the space-time curved;
- the motion of particles in a curved space-time occurs along the geodesic curves and reflects the influence of the gravitation on the dynamics of particles.

The space-time is curved in volumes of the space occupied by matter, but it becomes also curved in a vicinity of bodies due to the elasticity of the space-time. The equations for R_{ik} (the tensor of space-time curvature) were obtained by Einstein firstly in the form, where the source in these equations was only T_{ik} (the tensor of energy-momentum of matter). Then the equations were modified by the introduction of an additional source Λg_{ik} that is a cosmological term describing the antigravity:

$$R_{ik} - \frac{1}{2} g_{ik} R - \Lambda g_{ik} = \frac{8\pi G}{c^4} T_{ik}. \qquad (4.1)$$

In the middle of the 1960s, E. Gliner associated the Einstein cosmological term with vacuum, whose observed energy density ρ_V is determined by the cosmological constant Λ (see, e.g., [20–21]): $\rho_V = \dfrac{\Lambda c^4}{8\pi G}$. The value of Λ is not given by theory, and it can take any value that is consistent with experiment.

In the last decades, the cosmological consequences of the introduction of Λ were experimentally confirmed, and the following assertions are considered to be proved:

- Vacuum (dark energy) dominates in the Universe; by the energy density, vacuum exceeds all “ordinary” forms of matter taken together;
- dynamics of the cosmological expansion is guided by the antigravity;
- cosmological expansion accelerates, and the space-time becomes, in this connection, static.

In the worls by E. Gliner, the processes of accelerated expansion of matter were first connected with the antigravity of vacuum, and the creation of matter with quantum fluctuations of vacuum, which are caused by the acceleration. *Vacuum should be considered as a medium occupying all the space uniformly with good reliability from the cosmological scales down to centimeters.*

The equation of state of vacuum, i.e., the connection between the pressure p_V and the energy density ρ_V,

$$p_V = -\rho_V, \qquad (4.2)$$

follows from the theory of quantum fields and the thermodynamical reasoning. Let us use the thermodynamical identity $dW_V = TdS - p_V dV$ and represent the total internal energy of vacuum in the form $W_V = \rho_V V$. For the adiabatic processes in the homogeneous vacuum, $dS = 0$, and $dW_V = \rho_V dV$. Hence,

$p_V = -\rho_V$. By the Friedman theory [46], the gravity is created not only by the density of a medium, but also by its pressure according to the relation $\rho_{eff} = \rho + 3p$. For vacuum, the density of its effective gravitational energy $\rho_G = \rho_V + 3p_V = -2\rho_V$ is negative for a positive density. In connection with the unique equation of state (4.2) (see [21]), vacuum possesses several important properties that distinguish this medium among all others:

1. This medium cannot serve as a reference system. If there are the reference systems moving relative one another, then vacuum with the equation of state (4.2) accompanies every reference system. Hence, the nonaccelerated motion and the rest relative to this medium cannot be distinguished.
2. The medium with the equation of state (4.2) is unvariable and eternal. Its energy is the absolute minimum of the energy contained in the space.
3. The medium with the equation of state (4.2) creates the antigravity.
4. Vacuum creates a force, but it does not undergo (as a macroscopic medium) any action of external gravitational forces or the own antigravity (because the densities of the inertial mass $\rho_i = \rho + p$ and the gravitational mass of vacuum $\rho_G = \rho_i$ are equal to zero).
5. Vacuum is a medium uniformly filling the space on all scales from cosmological to small (by the data of modern experiments, down to scales of the order of centimeters). Experimentally, some manifestations of an inhomogeneity of vacuum were observed at the creation of nonhomogeneities of the medium on scales of the order of one micron and less (Casimir effects).

By virtue of the above-presented properties, vacuum plays the key role not only for the gravity, but also for any mass force, by revealing itself only in the noninertial accelerated reference systems. Therefore, substantiated is the assumption that the most important role in the interaction with vacuum is played by the electromagnetic field (vector potential) and the fields of negentropy (information) that transfer momenta to particles through the appropriate perturbations of the probability density distribution of particles in the space and create noninertial reference systems.

The violation of the condition of adiabaticity of the equation of state of vacuum on macroscopic scales of the order of meters or centimeters or less with the help of electromagnetic and entropic drivers will allow one to control its properties on these scales and to pose the question about its implication in energetic processes.

Changes of the entropy and the energy density on the scale of a perturbation of vacuum appear due to the action of mass forces and, hence, changes of the impactness:

$$\Delta S = \frac{\partial S_V}{\partial q}\delta q \approx \alpha_I \frac{\partial S_V}{\partial q}\delta Q_{imp}, \qquad \delta\rho_V = \left(T\alpha_I \frac{\partial S_V}{\partial q}\delta Q_{imp}\right)\rho_V. \tag{4.3}$$

4.1 Resonances at the interaction of longitudinal waves with vacuum

All main properties of vacuum are qualitatively determined from the equation of state and the uncertainty relation. Namely this relation reflects the peculiarities of vacuum, since it does not allow the conjugated quantities (e.g., a momentum and a coordinate or an energy and e time interval) to have simultaneously some exactly determined independent values. In this connection, the vacuum state cannot have the zero value of energy density, though it is defined as the state with minimal energy. The fluctuations of the vacuum state energy exist always, and it is impossible to get rid of them.

In a simple one-dimensional model, the fluctuational oscillations of vacuum are a collection of ideal oscillators with all frequencies. The energy density of elastic oscillations of vacuum with any frequency ω is

$$W_\omega = \frac{\langle p^2 \rangle / m}{2} + \frac{m\omega^2 \langle x^2 \rangle}{2},$$

where x is the coordinate, and $p = mu$ is the corresponding momentum at oscillations of an oscillator with effective mass m. Considering the formula for the energy, as the arithmetic mean of two terms, we obtain a chain of inequalities

$$\begin{aligned} W_\omega &= \frac{\langle p^2 \rangle / m}{2} + \frac{m\omega^2 \langle x^2 \rangle}{2} \geqslant \sqrt{\omega^2 \langle p^2 \rangle \langle x^2 \rangle} \\ &= \omega \left(\sqrt{\langle p^2 \rangle} \cdot \sqrt{\langle x^2 \rangle} \right) \geqslant \frac{\hbar\omega}{2}, \end{aligned} \tag{4.4}$$

where we use the uncertainty relation: $\Delta x \Delta p_x \geqslant \frac{\hbar}{2}$ on the last stage.

It follows from (4.4) that the energy minimum for oscillations of the oscillator turns out to be $W_{\min} = \frac{\hbar\omega}{2}$. The total energy density of all oscillations W_0 is equal to the integral contribution of all real frequencies from zero to infinity and, naturally, is infinite. Let us introduce a large, but finite scale L along a separate direction. Then the continuous set of frequencies becomes a discrete infinite sequence $\omega_n = n\frac{\pi c}{L}$, and $W_0(L) = \pi \frac{\hbar c}{2L} \sum_{n=1}^{\infty} n$.

Under the action of mass forces, the adiabiticity of vacuum can be broken, and an inhomogeneity l_R can appears. It will lead obviously to resonances due to a change of boundary conditions. As a result of the appearance of resonances between an electromagnetic field and vacuum, the infinite discrete sequence ω_n is separated from the continuum of frequencies. In this case, the total energy density of vacuum is infinite as before, but it is equal now to the infinite sum over all discrete frequencies $\omega_n = n\frac{\pi c}{l_R}$ or over all wavelengths $\lambda_n = \frac{\pi c}{\omega_n}$ (with regard for the resonance conditions $\lambda_n = \frac{l_R}{n}$):

$$W_0(l_R) = \pi \frac{\hbar c}{2l_R} \sum_{n=1}^{\infty} n. \tag{4.5}$$

Thus, the appearance of the space-time curvature causes a change of the energy:

$$\Delta W = W_0(L) - W_0(l_R)\frac{l_R}{L} = \sum_{n=1}^{\infty} \left(\frac{\pi\hbar c}{2l_R} n \right) - \sum_{n=1}^{\infty} \left(\frac{\pi\hbar c}{2l_R} \frac{l_R^2}{L^2} n \right). \tag{4.6}$$

The subsequent calculations are carried on within a regularizing procedure, which allows us to find the infinite sums with the help of the introduction of the efficient "cutting" of high harmonics and the use of the relations

$$\begin{gathered}\sum_{n=1}^{\infty} W_n \to \lim_{\lambda\to 0} \sum_{n=1}^{\infty} \exp\left(-\lambda W_n\right) W_n, \\ \sum_{n=1}^{\infty} n \exp\left(-xn\right) = \frac{1}{4sh^2\left(x\right)} \underset{x\to 0}{=} \left(\frac{1}{x^2} - \frac{1}{12} + \frac{x^2}{240} + \ldots.\right).\end{gathered} \tag{4.7}$$

First, we calculate difference (4.6) with a finite parameter λ. Then, by passing to the limit, we obtain the formula for the difference of energies in the one-dimensional case:

$$\Delta W \approx -\frac{\pi}{24}\frac{\hbar c}{l_R}. \tag{4.8}$$

In the three-dimensional case, the similar calculations were first performed by Casimir [22], who considered two plane surfaces with area S_{surf}, which are placed at the distance d from each other, and obtained th formula

$$\frac{\Delta W}{S_{surf}} \approx -\frac{\pi^2 \hbar c}{720 d^3} \tag{4.9}$$

and, respectively,

$$F_R/S_{surf} \approx -\frac{\pi^2 \hbar c}{240 d^4} \tag{4.10}$$

for forces acting on the unit area of a plate (Casimir forces).

The longitudinal electromagnetic waves with wavelength λ that realize the coherent acceleration $a_{cog} \approx \alpha_d \dfrac{u_T}{\tau_{eff}} \approx 8\alpha_d \dfrac{u_T}{\tau} \approx 8\alpha_d \dfrac{u_T}{\lambda/_C} \approx 8\alpha_d \dfrac{c u_T}{\lambda}$ of the separated subsystem of particles induce, in correspondence with (3.13), the space-time curvature with characteristic scale l_R:

$$l_R \approx \frac{c^2}{\sqrt{2} a_{cog}} \approx \frac{\lambda}{\sqrt{2}\cdot 8 \cdot \alpha_{d0}\left(A\right)\beta_T}, \qquad \alpha_{d0}\left(A\right) = \frac{\Delta p}{p_T} \approx \frac{e\cdot A}{p_T}. \tag{4.11}$$

Using the resonance conditions for wavelengths $\lambda_n = \dfrac{l_R}{n}$ and formula (4.9) for the space-time curvature scale, we obtain the formula for the resonance frequencies:

$$\omega_{0n} \approx \frac{1}{n}\frac{4\sqrt{2}\pi}{c} a_{cog} \approx \frac{1}{n} 2^{3/2} \beta_T \alpha_d \left(\frac{2\pi}{\tau_{eff}}\right) \approx \frac{2^{3/2}\beta_T}{n} \alpha_d \omega_0. \tag{4.12}$$

The coefficient of domination is proportional to the amplitude of the electromagnetic field in the medium and must take the growth of the amplitude at the resonance interaction of the field with the medium into account. At a resonance, the frequency dependence of the amplitude is as follows:

$$\alpha_d \approx \frac{\alpha_{d0}}{\sqrt{\left(1-\dfrac{\omega^2}{\omega_{0n}^2}\right)^2 + \left(\dfrac{\delta_{eff}}{\omega_{0n}}\right)^2}} \approx \frac{\alpha_{d0}}{\sqrt{\left(1-\dfrac{\omega^2}{\omega_{0n}^2}\right)^2 + \dfrac{\omega^2}{4\omega_{0n}^2}\dfrac{1}{Q^2}}}. \tag{4.13}$$

Here, δ_{eff} is the damping coefficient, $\Delta\omega$ is the width of a resonance, and $Q \approx \frac{\omega_0}{\Delta\omega}$ is the quality of a resonance. From whence and (62), we obtain the formula for frequencies

$$\omega_{0n} \approx \frac{2^{3/2}\beta_T}{n} \frac{\alpha_{d0}}{\sqrt{\left(1-\frac{\omega^2}{\omega_{0n}^2}\right)^2 + \frac{\omega^2}{4\omega_{0n}^2}\frac{1}{Q^2}}} \omega_0$$
$$\approx \frac{2^{5/2}\beta_T}{n}\alpha_{d0} Q\omega_0 \approx \frac{2^{5/2}\beta_T}{n}\alpha_{d0} Q\omega_0,$$

which yields

$$\omega_{0n} = \left(\frac{c\tau_{eff}}{\sqrt{2}\alpha_{d0}\beta_T\delta_d}\right)^{1/3}\delta_d. \tag{4.14}$$

Hence, we can conclude that, for large values of the coefficient of domination α_{d0}, the resonance frequency shifts to the lower frequencies.

Curvature and impedance.

It was mentioned in the previous section that, near a growing crystal and in the region of phase transitions or in the noninertial reference system in a more general case, the refractive index (or, in other words, the impedance directly connected with this physical quantity) is changed in the space. In the works by Podosenov (see [29]), the influence of constraints in electrophysical systems on the radiotechnical (electrophysical) elements entering their composition such as capacities and inductances were analyzed in details.

As was discussed above, vacuum in a noninertial system can be represented by a countable number of ideal fluctuating oscillators which can be modeled by circuits including capacities and inductances. These oscillators depend on the space-time curvature.

For a spherically symmetric motion of a system of charged particles with constraints, the Riemann space-time metric is determined by the acceleration $a_0 = \frac{eE}{2m}$, where $E \approx -\frac{\partial A}{\partial t}$ is the intensity of a longitudinal electric field on the surface of a sphere. The metric takes the form

$$ds^2 = \exp(\nu)\left(dy^0\right)^2 - \exp(\lambda)\, dr^2 - r^2\left(d\theta^2 + \sin^2(\theta)\, d\phi^2\right), \tag{4.15}$$

where

$$\exp(\nu) = \left(1 + \left(\frac{\kappa}{2}\right)^{1/2} d\right)^2, \qquad \exp\left(\frac{\lambda}{2}\right) = -r^2 \frac{\left(\frac{\kappa}{2}\right)^{1/2} d}{1 + \left(\frac{\kappa}{2}\right)^{1/2} d}. \tag{4.16}$$

The dependences of capacities and inductances on the space-time curvature κ were studied in [29]. In particular, the capacity $C\left(l_{eff}, \kappa\right)$ with characteristic spatial size l_{eff} and with characteristic surface area S_{eff} was obtained as

$$C\left(l_{eff},\kappa\right)\approx C_0\frac{\left(\frac{\kappa}{2}\right)^{1/2}l_{eff}}{1-exp\left(-\left(\frac{\kappa}{2}\right)^{1/2}l_{eff}\right)}$$
$$\approx C_0\left(1+\left(\frac{\kappa}{2}\right)^{1/2}l_{eff}\right)\approx C_0\left(1+\frac{a_{cog}}{2c^2}l_{eff}\right), \tag{4.17}$$
$$C_0=\varepsilon_l\frac{S_{eff}}{4\pi l_{eff}}.$$

We now determine a change of the impedance $\Delta Z(\kappa)$ of an oscillatory circuit with the capacity C and the inductance L at the given frequency ω near a resonance at a change of the space-time curvature κ. We take into account that the active part of the impedance tends to zero. Hence, we can approximately write

$$\Delta Z(\kappa)\approx\frac{1}{2}\left(\omega\frac{\partial L}{\partial\kappa}+\frac{1}{\omega C(\kappa)}\frac{\partial C}{\partial\kappa}\right)\Delta\kappa\approx\frac{1}{\omega C(\kappa)}\frac{\partial C}{\partial\kappa}\Delta\kappa\approx\frac{l_{eff}}{\omega\kappa^{1/2}}\Delta\kappa. \tag{4.18}$$

The significance of relations (4.17) even for small values of currents and voltages consists in that the devices including these electrophysical elements become basically nonlinear objects such as parametric oscillatory systems. At the certain choice of the excitation frequencies, such effects as an extension of the spectrum and the amplification of a signal can be revealed.

On the other hand, a more important circumstance can possibly consist in that a deviation of the impedance from the values determined by the capacity and the properties of a dielectric in the frame of linear electrodynamics can serve a measure of the space-time curvature.

On the basis of his theory of the time as a physical quantity possessing a density [27], Kozyrev constructed a very exact device (Kozyrev's gage) to measure the changes in the time density with the use of a bridge scheme consisting of resistors and a sensitive galvanometer. By essence, the changes of the time flow in the space-time are inseparably connected with a change of its curvature. In our experiments, we used a modified scheme of Kozyrev's gage with amplifiers instead of a sensitive galvanometer (see Fig. 4.1).

In the experiments carried out by Kozyrev with the use of its gage, one of the resistors of a balanced bridge served as a detector of the time flow and can be placed at various points of the region under study. A change of the impedance of this resistor was at once registered with a galvanometer. Kozyrev's gages were used as a tool in astrophysical studies by Kozyrev himself [27] and by other researchers [28].

In our studies, Kozyrev's gage was used as a sensitive meter of the space-time curvature and the appropriate resonance phenomena described above at the interaction of longitudinal electromagnetic fields with vacuum.

We have carried out the experiments on measuring the space-time curvature with the use of electromagnetic fields. As a source of longitudinal eelctric fields, we took the toroidal coil with a winding, on which a high-frequency current was flowing (see Fig. 4.2).

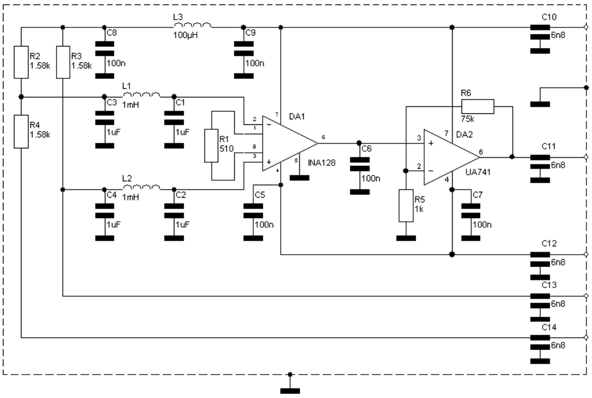

Fig. 4.1. Kozyrev's gage with operational amplifiers instead of a galvanometer

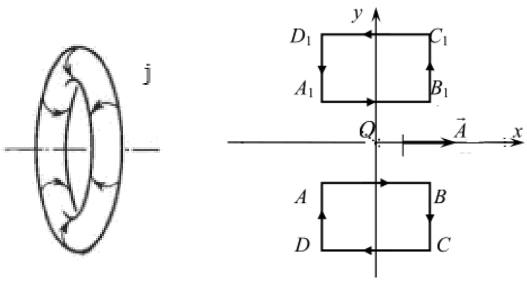

Fig. 4.2. Toroidal coil as a source of the variable vector potential and, as a consequence, of a longitudinal electric field.

As was expected, the maximal values of the rate of variation of the vector potential were observed on the axis of the toroidal coil.

The amplified signal from the bridge, which was proportional to a change of the impedance in the studied region where we mounted a detector, was supplied to a computer through an analog-digital unit. The changes of the impedance and, hence, the curvature were essentially different at different frequencies of a longitudinal field. The resonance frequencies were clearly distinguished.

The switching-on of a generator was naturally accompanied by an increase of the temperature in the region, where Kozyrev's gage was located. In Fig. 4.3, we show the time dependence of the measured voltage on the bridge, as well as the time dependence of the temperature.

The plots indicate clearly the complete absence of correlations of the temperature and the readings of a gage. At the time moment of the switching-on of a current in the coil, we observed a sharp change of the impedance. At the switching-off of the current, the values of signals from a bridge approach the initial values, whereas the temperature varies significantly slower.

The amplitude of variations of the impedance and, hence, the curvature demonstrated a strong dependence on the frequency of a current flowing along the emitter. The resonance behavior of the amplitude as a function of the frequency is shown in Fig. 4.5.

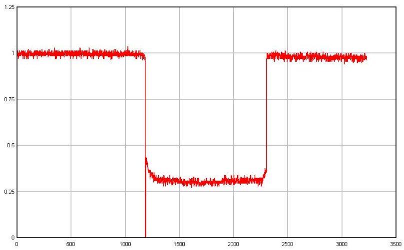

Fig. 4.3. Temporal dependence of a signal from Kozyrev's detector. The signal is proportional to a change of the impedance and the space-time curvature.

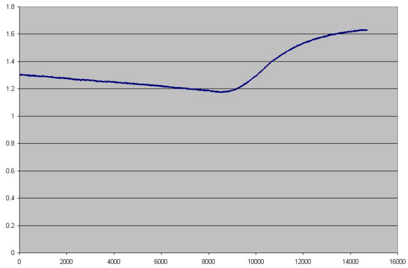

Fig. 4.4. Behavior of the temperature during the measurement.

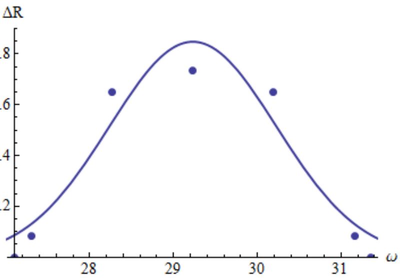

Fig. 4.5. Resonance curve for one of the resonances in the region of frequencies of the order of 29 MHz.

The values of the resonance frequency and the quality of resonances observed in experiments are in good agreement with (4.11) and (4.12).

4.2 Regularized wave equations as a model of vacuum

The physical vacuum, as a medium with specific properties, interacts with particles located in it. This interaction can accelerate particles or decelerate them. In the last case, we can say that the particles moving with acceleration undergo the action of friction due to the fluctuations of vacuum.

As was shown above (Section 2), the system of particles evolves mainly in NRS, and the efficient driver of mass forces initiating NRS is an electromagnetic driver. Below, we will study some peculiarities of the evolution of a system of charged particles at the interaction with vacuum under the action of electromagnetic fields with electric field intensity $\vec{E} = -\dfrac{\partial \vec{A}}{\partial t} - \nabla\varphi$. Only the first term in this formula is responsible for the initiation of mass forces in the system (see Section 2), since only this term represents the electric field inside a homogeneous system. We consider the action of the first and second terms on charged particles as the mass force $\vec{F}_m = -e\dfrac{\partial \vec{A}}{\partial t}$ and the Coulomb force $\vec{F}_c = -e\nabla\varphi$, respectively.

It is known [47] that the account for collisions between charged particles (electrons) in the approximation with the Landau collision integral allows one to describe the appearance of the friction force (from the side of ions), which rapidly decreases with increase in the velocity of electrons $\vec{F}_{fr} \approx -\frac{m_e}{4\pi} L^{e/i} \frac{n_i}{u^3} \vec{u}$ for velocities larger than the thermal one. In this case, it is significant that the friction force has a maximum over velocities, $\max F_{fr} \approx -0.2 \frac{e^2}{r_D^2}$, after the averaging.

For low fields, the condition of domination (2.16) does not hold. Then the charged particles obey the phenomenological equation of charge transfer or the equation of motion (τ_{eff} is the effective duration of the momentum transfer in collisions) in terms of ordinary derivatives:

$$m_e \frac{d\vec{u}}{dt} = e\vec{E}_c - \frac{1}{\tau_{eff}} (m_e \vec{u}) . \tag{4.19}$$

For a quasistationary state of charged particles from the noncoherent part that are characterized by a constant velocity and satisfy the condition

$$e\vec{E}_c - \frac{1}{\tau_{eff}} (m_e \vec{u}) \approx 0,$$

Eq. (4.19) yields the Ohm law for the current density $\vec{j} = \rho_e e \vec{u}$:

$$\vec{j} = \sigma_E \vec{E}_c, \qquad \sigma_E = \frac{\rho_e e^2 \tau_{eff}}{m_e}. \tag{4.20}$$

The presence of homogeneous longitudinal fields in a system of charged particles satisfying condition (2.16) corresponds to the appearance of mass forces $\vec{F}_m$. In fact, the condition of domination of the action of an electromagnetic driver (2.16) corresponds to the presence of an electromagnetic force in the system particles, which exceeds the critical friction force ($\vec{F}_m > \max \vec{F}_{fr}$). A part of charged particles (particles forming a coherent subsystem), whose share is equal to the order parameter η, are unboundedly accelerated. Another part of particles $(1 - \eta)$ turns out noncoherent, is decelerated by the friction force $\approx -\frac{1}{\tau_{eff}} (m_e \vec{u})$, and obeys the equation with ordinary derivatives (4.19).

In the system, two components appear in the general case: the coherent component with density ρ_{cog} and the corresponding velocity u_{cog} and the noncoherent one with density ρ and velocity u. The ratio of components determines the order parameter $\eta = \frac{\rho_{cog}}{\rho_{cog} + \rho}$, and the velocities of the components satisfy different equations of motion.

The differential equations with ordinary derivatives (Riemann derivatives), which were used for the description of the processes of transfer, are based on the fact that the translations in the space-time are characteristic ttransformations for IRS.

The satisfaction of the condition of domination (2.16) and, hence, the transition into a state with sharply decreasing resistance correspond to the appearance of a motion of the coherent part of the system as the whole with explosively increasing drift velocity (in other words, to the formation of NRS). In NRS for the systems in coherent states, the characteristic features are the presence of many scales and the self-similarity of the processes of evolution, which is reflected in a complicated (fractal, in the general case) structure of the space-time. In this case, the use of alternative definitions of the operators of differentiation, which appear due to the regularization, for the description of the dynamics of a physical situation seems to be more adequate [30].

As was shown above, the physical vacuum under conditions of the action of mass forces is characterized by a discrete set of frequencies and, hence, scales of the time. The coherence appearing in vacuum can possess the properties of similarity (fractal properties). In this case, the Jackson derivative is the most natural generalization of the notion of derivative for the description of the evolution of all quantities with the properties of similarity [31]. Let us consider the definition of this derivative, which is used, in particular, for the determination of the rate of processes. The operator of shift is replaced by the operator of scaling (with the coefficient of similarity q_s) passing in the limit into the ordinary derivative D_t:

$$D_{q_s} f\left(t\right) = \frac{f\left(q_s t\right) - f\left(t\right)}{q_s t - t}, \qquad D_t f\left(t\right) = \lim_{q_s \to 1} D_{q_s} f\left(t\right). \tag{4.21}$$

The eigenfunction of the Jackson operator is the scaling generalization of the exponential function, namely

$$e_{q_s}^{t} = \sum_{k=0}^{\infty} \frac{t^n}{[k]_{q_s}!},$$

which satisfies the relation $D_{q_s} e_{q_s}^{t} = e_{q_s}^{t}$. Here, the Jackson q-number

$$[n]_{q_s} = \frac{q_s^n - 1}{q_s - 1} = q_s^{n-1} + \ldots + 1.$$

The coherence of a state of the system (scaling invariance) is revealed, naturally, in the oscillatory processes. It is easy to verify that the functions that depend on the scaling parameter q_s and are defined by the relations

$$\cos_{q_s}\left(z\right) = \frac{e_{q_s}^{i\,z} + e_{q_s}^{-i\,z}}{2}; \qquad \sin_{q_s}\left(z\right) = \frac{e_{q_s}^{i\,z} - e_{q_s}^{-i\,z}}{2i}, \tag{4.22}$$

satisfy the relations characteristic of ordinary trigonometric functions and are the solutions of the equation for a fractal oscillator with Jackson derivatives. These generalized scaling functions pass into ordinary trigonometric functions as $q_s \to 1$. Respectively, the difference between the former and the latter increases with the deviation of the scaling parameter from 1.

The deviation of the parameter of similarity q_s from 1 reflects a degree of openness of the system, despite the absence of an explicit dissipative term in the equation. Now, the openness of the system is characterized by the indices of differential operators of quantum analysis, rather than the parameters of dissipation. In open systems, the oscillatory processes are dissipative for the parameter of similarity $q_s < 1$ or are unstable for $q_s > 1$ (see Appendix 3).

The parameters of similarity q_s, nonequilibrium q, and damping δ are connected by the relations that can be found from the condition of maximal coincidence of phase trajectories in the quadratic metric. The result of such an optimization in the region of values of the parameter of similarity $0.7 < q_s < 1.5$ give the function (see Appendix 3):

$$q\left(q_s\right) = \begin{cases} 2.023 - 1.5608 q_s + 0.5380 q_s^2, & q_s \leqslant 1 \\ 1.7005 - 0.9234 q_s + 0.2223 q_s^2, & q_s > 1 \end{cases}. \tag{4.23}$$

Let us consider the influence of the coherence of a state on the processes of transfer of charged particles in the physical vacuum under the action of mass forces. More exactly, we will obtain a generalization of the Ohm law, which will be valid for the coherent states with the coefficient of similarity q_s in homogeneous longitudinal fields $\vec{E} = -\dfrac{\partial \vec{A}}{\partial t}$ created by a nonstationary vector potential $A\left(t\right)$.

It is clear that, with the use of the scaling transformations, the equation for the velocity of charged particles u in the case under study can be written in the form

$$D_{q_s} u = -\frac{e}{m}\left(\frac{\partial A}{\partial t}\right). \tag{4.24}$$

Acting by the integral Jackson operator $\hat{I}_{q_s} \equiv D_{q_s}^{-1}$ (see Appendix 3) on both sides of this relation, we obtain $u = \dfrac{e}{m}\hat{I}_{q_s}\left(-\dfrac{\partial A}{\partial t}\right)$. From whence, we arrive at the relation between the current density $j = e\rho_e u$ and the vector potential $A\left(t\right)$:

$$j \approx \frac{\rho_e e^2}{m\tau_{eff}}\hat{I}^v\left(-A\left(t\right)\right). \tag{4.25}$$

The proposed model of the phenomenon of transfer and oscillatory processes in fractal media on the basis of the apparatus of quantum derivatives can be a mathematical foundation for the development of new radiophysical devices using the specific properties of nonlinearity and irreversibility of the fluctuations of vacuum in NRS.

We now consider the alternative phenomenological description of the motion of the coherent part of a system of charged particles without the use of quantum operators, but with the direct application of the interaction with vacuum in NRS. As was shown above, the friction of permanently accelerating particles satisfying the condition of domination caused by collisions with other particles can be neglected. However, by virtue of the fact that these charges move as the whole and form NRS, the force of their interaction with the physical vacuum turns out to be nonzero.

The situation is similar to the motion of a body in the ideal fluid. The motion of a body with constant velocity occurs freely, and the body does not feel the presence of the medium (see the d'Alembert paradox). The motion with acceleration leads to the appearance of an associated mass and the interaction with the medium, which is proportional to the acceleration.

The motion of particles in the physical vacuum subordinates the analogous laws. The motion with acceleration leads to the interaction with vacuum and the appearance of forces $\vec{F}_{vac} = \delta_{vac}\dfrac{d\vec{u}}{dt}$. In this case, the equation of motion of the coherent part takes the form

$$m\frac{d\vec{u}}{dt} = -e\frac{\partial\vec{A}}{\partial t} + \vec{F}_{vac} \quad \text{or} \quad (m-\delta_{vac})\frac{d\vec{u}}{dt} = -e\frac{\partial\vec{A}}{\partial t}, \tag{4.26}$$

which yields the Ohm law for the coherent part of charged particles:

$$\vec{j} = \sigma_A\vec{A}, \qquad \sigma_A = \frac{\rho_e e^2}{(m-\delta_{vac})}. \tag{4.27}$$

It is seen that the obtained Ohm law coincides with the London equation [48]. On the whole, the model of the interaction of particles with vacuum under the action of mass forces is similar to the two-fluid model of superconductivity: in the coherent state, the currents of particles arise in the absence of the difference of potentials.

The value of mass defect δ_{vac} at the interaction of particles with vacuum is determined by the explosive local expansion of the space-time with a curvature corresponding to the resonance frequencies (4.10) in metric (3.12).

4.3 Coherent acceleration of the reference system and criteria for the initiation of a collective synthesis

As was shown above, the basic physical quantity that initiates the processes of synthesis in an ensemble of particles in correspondence with the principle of dynamical harmonization is the coherent acceleration of this ensemble of particles.

Let us find the conditions for the acceleration of NRS that will ensure the intiation of MQO from the strongly nonequilibrium shell—i.e., the condition for the initiation of a instability leading to the reconstruction of a state of the system such as the phase transition from the stable neutral substance to a quasineutral electron-nucleus plasma. For the first time, the conditions for the appearance of a positive feedback with respect to the density in a plasmoid were found by A. Vlasov in the frame of his nonlocal kinetic theory [15]: *"The binding energy is released at a decrease of the radius of the formation and turns out to be sufficient for the support of the processes of ionization. The mechanism of the processes of ionization consists in the creation of intrinsic inhomogeneous electrostatic fields, which is a consequence of the oscillatory change of the potential of interaction of ions in the space through an intermediate system".*

As is clear from the above, the efficient model for the description of the action of mass forces in the system of many particles is given by the Schrödinger equation and the de Broglie–Bohm representation of it in the form of a system of equations for real functions. The use of the Dirac equation for an electron in the Coulomb field of the kernel and its reduction to the Schrödinger equation *allows us to describe the mechanism and the conditions for the self-ionization of an ion on the basis of the initiation of the processes of collapse of electrons with regard for relativistic corrections* (see [5, 49–50]).

The conditions of stability of a dense substance (e.g., a metal) are mainly determined by properties of a degenerated electron gas. The ionization equilibrium can be changed, by varying the thermodynamical parameters such as the temperature and the density.

The increase of the degree of ionization of a substance Z due to a decrease of the number of electrons shielding the nucleus causes a decrease the corresponding radius of outer electron shells.

A decrease of the size of these shells due to a change of the Coulomb repulsion leads to a decrease of the equilibrium distance between ions and, hence, to a growth of the density as compared with that in the stable state ρ_{stabZ}:

$$\rho_{stabZ} \approx 3.784\, Z^{2.049}. \tag{4.28}$$

The increase of the density of a substance alone can cause the ionization (figuratively, we may say that the pressure “crushes” and breaks the outer electron shells). However, this process requires very high critical densities,

$$\rho_{critionZ} \approx 5.\, 10^2 Z^2, \tag{4.29}$$

and the appropriate pressures. These critical densities are approximately by two orders larger than those appearing at the increase of the degree of ionization by 1. The ratio of densities ρ_{crion}/ρ_{stab} as a function of the ion charge Z is shown in Fig. 4.6.

Thus, the instability in the ordinary state does not arise, since the increase of the density due to the previous stage of ionization is unsufficient for the further increase of the ion charge, which ensures the stability of the substance arrounding us relative to its spontaneous collapse under equilibrium conditions.

There are the external actions on a system, at which this stability is broken. In connection with that the compression of a substance is hampered by the repulsion of like charges, all physical situations that ensure a decrease of the Coulomb repulsion due to the renormalization of the Coulomb interaction increase the equilibrium density of a substance and can induce the loss of stability.

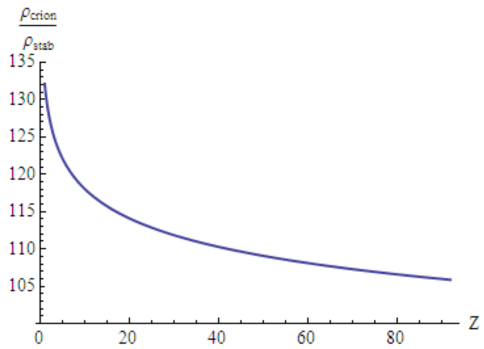

Fig. 4.6. Ratio of the density leading to the increase of the ion charge by 1 to the density orresponding to the current ion charge.

The main contribution to the conditions of equilibrium is given by the energy of degenerate electrons. The system of particles in a thin layer (shell) becomes nonequilibrium even only due to the geometry, since two basicall different states of motion can be realized:

- perpendicularly to the thin plasma layer, the motion is bounded and, hence, has a clearly expressed discrete energetic structure;
- the motion along the layer is not bounded and has, hence, the continuous values of energy.

In order to initiate the positive feedback leading to the collapse, it is necessary to decrease the critical density approximately by 5.102/3.784 = 132 times or to delocalize the ion approximately by a factor of 5.1. Such values are available if the order parameter satisfies the following condition:

$$\eta \geqslant 0.5\left(1-\left(\frac{1}{5.1}\right)^{1/\gamma_R}\right) \approx 0.35 \quad \text{or} \quad q > 1.54. \tag{4.30}$$

The satisfaction of the conditions for existence of a positive feedback leads to the explosive process of ionization and to the appearance of an electron-nucleus plasma with the density

$$\rho_{en} \approx \frac{10}{m_p} Z_l^4 \text{ cm}^{-3}. \tag{4.31}$$

At such a density, the mean distance between nucleons R_{av} and the characteristic size of nuclei R_{nuc} in a fluid are, respectively,

$$R_{av} = \left(\frac{3}{4\pi\rho_{en}}\right)^{1/3}, \qquad R_{nuc} = 1.2\, 10^{-13} A_l^{1/3}. \tag{4.32}$$

Let the pressure in the environment be p_0. Then the collapse time of a shell can be estimated by the Rayleigh relation (the more general Zababakhin relation can be used as well):

$$t_{ex} \approx 0.9 R_{sh}\sqrt{\frac{\rho_l}{p_0}} \approx 9.\, 10^{-7} R_{sh}\sqrt{\frac{\rho_{lg}}{p_{0atm}}}. \tag{4.33}$$

The behavior of the radius tending to zero is determined by the relation

$$R_{sh} = R_{sh0}\left(1-\frac{t}{t_{ex}}\right)^{\alpha_m}, \qquad \alpha_m \approx \frac{(1+\kappa)}{2}, \quad 0<\kappa<\infty. \tag{4.34}$$

Here, ρ_{lg} is the density of the environment in gr/cm3, p_{0atm} is the external pressure in atm, and the shell radius in cm.

For the interaction characteristic of Maxwell molecules and hard spheres, $\kappa \approx 4/3$ and $\alpha_m \approx 7/6$. The analysis of solutions of the equations of dynamics of a shell yields the "scaling" relations between characteristic macroscopic scales of shells (between radii and thicknesses) and their mean densities at two time moments:

$$\begin{aligned} &\frac{R_2}{R_1} \approx \left(\frac{\rho_1}{\rho_2}\right)^{3/14}, \quad \frac{d_2}{d_1} \approx \left(\frac{\rho_1}{\rho_2}\right)^{4/7} \\ \text{or} \quad &\frac{\rho_1}{\rho_2} \approx \left(\frac{R_2}{R_1}\right)^{14/3}, \quad \frac{d_2}{d_1} \approx \left(\frac{R_2}{R_1}\right)^{8/3}. \end{aligned} \tag{4.35}$$

Thus, the density at the collapse of a shell increases explosively, and the shell thickness decreases. Their behavior is described by the relations:

$$\rho_{sh} \approx \frac{\rho_0}{\left(1-\dfrac{t}{t_{ex}}\right)^{14\alpha_m/3}}, \qquad d_{sh} \approx d_{sh0}\left(1-\frac{t}{t_{ex}}\right)^{8\alpha_m/3}. \tag{4.36}$$

The density corresponding to an electron-nucleus plasma ρ_{en} is attained at the time t_{en} given by the formula

$$t_{en} \approx \left(1 - \left(\frac{\rho_{en}}{\rho_0}\right)^{-\dfrac{3}{14\alpha_m}}\right) t_{ex}, \tag{4.37}$$

and the acceleration during the collapse increases with time.

The process of collapse of shells occurs under conditions of a dominating perturbation along the radius from the very beginning of the process (the acceleration of a coherent motion is more than the acceleration of the dissipation), whereas a decrease of the thickness acquires large accelerations only at the end of the process. At at the end of this stage, the acceleration of electrons is $a_{cog} \approx 10^{29}$ cm/sec^2, and the space-time curvature attains values of the order of 10^{18}, which corresponds to the atomic scale less than $10^{-9} \div 10^{-8}$cm.

The process of collapse in the electron-nucleus plasma, which is the fall of electrons in a Wigner–Seitz cell onto its Coulomb center (the nucleus with charge Z), is accompanied by the subsequent increase of the coherent acceleration up to the limitedly high values of the order of $a_{cog} \approx Z^2 \cdot 10^{29}$cm/sec^2. These accelerations can already ensure the space-time curvature to be more than 10^{22}, which corresponds to characteristic scales $\leqslant 10^{-11}$ cm.

The attained scales approaching the nuclear ones correspond to the high rates of change of the entropy gradient and ensure the flattening of the wave functions of all particles of the system and the formation of MQO with the scaling from the macro- down to nuclear scales.

Thus, the self-consistent ionization of a substance due to the loss of stability caused by the action of mass forces occurs explosively and is accompanied by a change of the number of constraints in the system. The explosive change of the entropy in time and space leads to the existence of accelerations of all orders. The mass force appearing in these processes in a self-consistent way causes the explosive "flattening" of the wave functions of nuclei. If the effective size of a nucleus tends to the mean distance between nuclei, then the order parameter approaches 0.5, and MQO is formed. The formation of MQO initiate the collective synthesis of new structures, whose efficiency depends on the dynamics of a coherent acceleration and, hence, on the space-time curvature.

5 Conclusions

The present work is a part of the cycle of works [1–2] devoted to the description of the theory of self-organization of the systems with varying constraints and the control over the synthesis. It is made in the frame of the development of the conception of self-organizing synthesis (see [1], [5]) on the basis of the principle of dynamical harmonization.

In the work, we have presented the geometric approach to the variational principle of dynamical harmonization, which allows one to solve the problems of self-organization and control over the directedness of the evolution of various complicated systems, basing on the single viewpoint from the very general positions of the theory of dynamical systems with varying constraints.

The comprehension of the geometric nature of physical laws was started by Clifford [51] and was developed by Hilbert, Einstein, and Wheeler [52–54].

In his mathematical works concerning the work by Riemann [55], Clifford wrote as early as 1876: "I consider that

1. Small parts of the space are really analogous to small hills on the surface, which is plane on the average, namely: the ordinary laws of geometry do not valid there.
2. The property of curavture or deformation continuously passes from one part of the space to another one like a wave.
3. Such a change of the space curvature reflects the real phenomenon called by the motion of matter, which can be the ether or a weighty substance.
4. Only such changes obeying (possibly) the law of continuity occurs in the physical world".

Einstein analyzed the gedanken experiment with particles in the field of mass forces [53] and made conclusion that the light velocity is changed in the field of gravitational mass forces, and, hence, the space-time curvature appears. In 1919, the phenomenon predicted by Einstein was discovered experimentally during Sun's eclipse.

We note that the single property of the field of gravity, which was used in the theoretical reasoning [53], was its mass character. The analysis of the principle of dynamical harmonization and the basic positions of the conception of self-organizing synthesis [1, 5] allowed us to generalize the idea of general relativity theory of the curved space-time in the field of gravitational forces to any mass forces.

The assertion that the mass forces of any nature (satisfying the condition of domination) decrease the light velocity and curve the space-time is basic for the geometrization of the theory of evolution and control. It is worth to mention that a decrease of the light velocity in the region, where the coherent acceleration is present due to the growth of crystals, was experimentally discovered much earlier (see [25]) than a decrease of the light velocity near massive gravitating bodies.

In the frame of the theory constructed by us, we have obtained the connection between a change of the space-time curvature in quasihomogeneous electromagnetic fields and a change of the impedance (see (4.16)), which was registered in the experiment with the help of Kozyrev's gage (see [27], [28]) in the modern version.

The important circumstance for the construction of the geometrodynamics of many-scale systems with varying constraints is the following: the most important notions joining all the scales are the space-time and the entropy (or information), and the mass forces of various nature act, as usual, on the own interval of scales, but ensure the nonlocality of processes.

The comprehension of the geometric nature of nonlocality allowed Vlasov to construct a nonlocal statistical theory [14–15], which is based on the geometry of support elements — the Finsler geometry [56]. The above-presented foundations of the geometrodynamics of evolution and control for the systems with constraints belong to the series of available theories (general relativity theory and nonlocal statistical theory).

As usual, the control over dynamical systems and the optimal synthesis of new structures is realized for the system, *whose state is set by the vector* *in the Euclid configurational space with a given matrix of the constraint coefficients.* In this case, the control that is a vector of forces acting on the appropriate components of the system can be optimized on the basis of the solution of a variational problem with *given functional.*

In the many-scale shell model of self-organization, the situation is significantly more complicated.

1. Evolution of the systems with varying constraints occurs in the Finsler space-time. The state of the system is set by the positions of particles in the anisotropic four-dimensional Riemann space-time (base space) and by their velocities, which are tangent to the trajectories of particles at a given point and, hence, belong to the corresponding layer of the tangent bundle of the space-time. The evolution of the system, i.e., the evolution of constraints of the system, runs also in the own layer of the space-time, where the coordinates characterize the structure of the system (such coordinates are, e.g., the fractal dimension of the system or its entropy);
2. On all stages of the process of synthesis, the evolution of systems obeys always the general variational principle for the systems with varying constraints, namely, the principle of dynamical harmonization. In the geometric statement, it asserts that the system evolves always along the geodesic lines in the Finsler space-time with regard for of the constraints in the system. In this case, the optimization functional is the space-time metric defining its curvature.
3. The defining role in the efficient control over the evolution is played by the coherent acceleration (in the general case, the tensor of accelerations) in the basic Riemann space-time. The current coherent acceleration in the basic Riemann space-time determines the constraints in a system (see (3.28)) and the evolution of the system in the tangent bundle (with the fractal dimension D_f as a structural coordinate in the layer) in agreement with the equations of dynamical harmonization in the Euler–Lagrange form with the corresponding Lagrange function

$$L_{str} = m_{str}\left(D_f\right) R_0 \frac{\dot{D}_f^2}{2} + sB_A\left(Z, D_f\right) A - U_{str}\left(D_f\right)$$

   (see (3.39)):

$$\frac{d}{dt}\left(\frac{\partial L_{str}}{\partial \dot{D}_f}\right) - \frac{\partial L_{str}}{\partial D_f} = 0.$$

4. Evolution of the system chabges the metric of the basic space-time (see (3.12), (3.27), (2.37), and (2.38)):

$$ds^2 = \left(dx^0\right)^2 - \sigma^2\left(x^0\right) g_{\alpha\beta}\left(x^1, x^2, x^3\right) dx^\alpha dx^\beta,$$

$$\sigma\left(x^0\right) = \exp_q\left(\frac{x^0}{c\tau_{eff}}\right),$$

$$q\left(\eta\right) = \begin{cases} q_- = 1-\eta, & q \leqslant 1 \\ q_+ = \dfrac{1}{1-\eta}, & q > 1 \end{cases}.$$

   Hence, we may assert that the order parameter η controls the space-time metric.

The control is realized by the external vector of controlling mass forces, which sets the contributions to the appropriate components of coherent accelerations of the system. The examples of mass forces that are the most important for the self-organization (evolution) are as follows:

1) forces of gravity and inertia;
2) entropic forces related to the entropy production gradient;
3) drift forces in a plasma involving the runaway of electrons in an electric field;
4) forces arising at the polarization of vacuum, including forces of the Casimir type.

It is possible to assert that the harmonized (nonforce) control creates the space-time curvature, which is necessary for that a configuration of the system and its state will “roll down,” like free ones, into the regions optimal for the realization of the process with a desired energy directedness.

By using the de Broglie–Bohm reprentation for the Schrödinger equation, we have shown the connection of nonlocality and coherence for the systems of many particles with the entropy production and mass forces. We have demonstrated that the entropic field is integral with the fields of constraints in any quantum system, in particular in MQO. Moreover, the introduction of entropic forces induces a nonlocality similar to the quantum one even in macroscopic systems. We have also analyzed the various means to create mass forces in the system and have obtained the relations for their calculation.

In a certain meaning, the space-time curvature is a hidden parameter. Since the separation of variables at the solution of the Schrödinger equation does not cause the disappearance of correlations between coordinates and momenta $r_{xp}(\kappa)$ (due to the curvature), the Schrödinger–Robertson uncertainty relation (3.25)

$$\Delta x \Delta p_x \geqslant \frac{\hbar}{2\sqrt{1 - r_{xp}^2(\kappa)}}$$

is valid and can be used for the control over many physical processes. For example, it would be used for the development of methods of a sharp increase of the transparency of Coulomb barriers and, hence, the probabilities of nuclear reactions [43].

The conducted studies allowed to generalize the Heisenberg uncertainty principle for energy and time in systems with variable constrains and thus with change of energy of constrains ΔE, so that this ratio is directly includes the entropy change of the system (i.e., a degree of openness (see 2.26))

$$\Delta t \Delta E \approx \frac{\hbar}{2} \Delta S.$$

It is now quite clear that the ratio of the classical and quantum properties of the system is determined not only by the value of the Planck’s constant $\hbar$, but also over the production of entropy in the system.

The developed theory of self-organization of open systems differs from the traditional nonequilibrium thermodynamics by the role of dissipation in the processes of evolution. Usually, the irreversibility of processes in a system is determined by the transition of the energy of a regular motion into the energy of a thermal random motion. In the theory with the principle of harmonization, the constraints and the structure of a system vary continuously at each hierarchical level, and the evolution is running without significant transition of energy into heat. It is obvious since one of the most important requirements to the external actions initiating the self-organizing evolution is the excess of the values of momenta of particles, which are formed by the controlling mass forces, over their thermal momenta in the system.

In the frame of the constructed geometrodynamics of the systems with varying constraints, the results obtained in works of the cycle substantiate theoretically all basic positions of the conception of self-organizing synthesis presented in [5]. Thus, the sequence of the basic processes at the evolution of the system can be presented as follows:

1. Separation of an ensemble of particles that will be evolved in the future.
2. Coherent acceleration of the ensemble and the formation of NRS as a result of the action of a dominating perturbation (creation, e.g., by electromagnetic or entropic mass forces).
3. Explosive self-consistent formation of MQO (usually of the “shell” type) when the coherent acceleration exceeds the threshold value.
4. The running of the processes of synthesis with energy directedness corresponding to the attained coherent acceleration (and, hence, to the attained space-time curvature),
5. Termination of the self-consistent process of evolution and the fixing (hardening) of products of the synthesis as a result of development of an instability on the very small scales.
6. Initiation and development of the explosive processes as a result of the release of the free energy of the synthesis of new structures.

The relations obtained in the theory of self-organization can be applied to the control over the optimal synthesis of systems with variable constraints of completely different nature and with different scale levels from nuclei and the interaction of particles with the physical vacuum to social and biological systems with complicated organization.

In many cases, a nuclear reaction is impossible because of the Coulomb repulsion of the nucleus. But the internuclear Coulomb barrier prevents only in the case when the distance between the nucleis is much greater than their de Broglie wavelength. If the de Broglie wavelength λ_{DB} for the nuleus is longer than the distance between the core nucleis, then the MQO in quantum multipart system is being formed, and as it shown in the work, the Coulomb barriers can effectively be decreased. The possibility of increasing of the de Broglie wavelength $\lambda_{DB} = \dfrac{2\pi\hbar}{|\vec{p} - \vec{p}_s|}$ (up to infinity) for each particle (monomer) of the ensemble, as seen, is associated with the presence of the entropy pulse p_s in them.

We emphasize once more that the base of the presented theory were main positions of the self-organizing synthesis of nuclei (see [5]), which allowed us to develop at once the means to initiate the nucleosynthesis by electron beams in a plasma diode [57]. The realization of this means allowed us to synthesize a wide spectrum of nuclei and their isomers (see [5, 2, 58–60]), and the use of electromagnetic drivers gave possibility to efficiently control the lifetime of radioactive nuclei. The experimental results concerning the electromagnetic control over the synthesis of nuclei and the rates of nuclear processes, as well as their comparison with the theory of self-organization of the systems with varying constraints, will be considered in the next article of the cycle.

The developed theory becomes rapidly a foundation for the creation of new technologies of the control over the synthesis, in particular, over the synthesis for the production of isomers-accumulators, and for the design of powerful environmentally safe "on-line" sources of nuclear energy.

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Appendix 1. Thesaurus of the self-organization of complex systems with varying constraints

Action. The action is the quantity

$$S = \int_{t_1}^{t_2} dtL\left(q, \dot{q}, t\right) \quad \text{or} \quad S = \int_{t_1}^{t_2} dt \left(\sum_i p_i \dot{q}_i - H\left(q, \dot{q}, t\right) \right),$$

where t — time, $q = \{q_1, \ldots, q_N\}$ — complete collection of coordinates characterizing the dynamical system (its configurational space), $\dot{q} = \{\dot{q}_1, \ldots, \dot{q}_N\}$ — collection of velocities (derivatives of q with respect to the time), L — Lagrange function depending on N coordinates, N velocities, and, sometimes, explicitly on the time. In classical mechanics, the action coincides with the difference of kinetic and potential energies; H — Hamilton function that is the total energy of the system depending on N coordinates, N momenta conjugated them, and, sometimes, explicitly on the time.

Bifurcation point — point of branching of possible ways of the evolution of a system. In the differential formalism, the solutions of nonlinear differential equations are branched at such a point.

Blow up:

- *Duration of the blow-up* — finite time interval, during which the process is developing with a superhigh rate.
- *Blow-up mode* — mode possessing a long-term quasistationary stage and a stage of superfast growth of the processes in open nonlinear systems. The dynamics of basic quantities in the blow-up mode is described by an explosive function $\approx \left(1 - \frac{t}{\tau}\right)^{-\nu}$ diverging at the blow-up time moment τ.

Coherence — from the Latin word “cohaerentia” — internal connection, connectedness. The behavior of elements inside the system that is consistent in time and space. In physics, it is the consistent running of several oscillatory or wave processes in time and space. Coherent behavior of elements — base for the appearance of space-time structures. Coherence is continuously connected with correlations of the basic quantities in the system.

Coherently correlated states. Coherently correlated states (CCS) are a complete collection of nonstationary states, in which the process of delocalization can be expanded. The equilibrium CCS usually used for the description of the systems weakly deviating from equilibrium ones (with small accelerations and flows). To describe the processes of delocalization with limitedly large accelerations, it is necessary to use the expansions in the eigenstates of systems that are in strongly nonequilibrium states, namely nonequilibrium CCS.

In quantum mechanics, coherent states are states with minimal disoersion (states with the probability distribution in the form of a Gauss distribution), i.e., they are states that are the closest to macroscopic states of the system.

Dimension of a system:

- *Dimension of the embedding of a system* — minimal number of parameters completely describing a state of the system.
- *Fractal dimension* — fractional dimension characterizing the self-similarity and the scaling invariance of systems.

Dissipation — processes of energy dispersion, its transformation in less organized forms (heat) as a result of dissipative processes such as heat conduction, diffusion, etc.

Dominating perturbation — mass force creating the coherent acceleration of particles of a system and, hence, a flow in the phase space. The value of constant flow in the phase space determines the dominating perturbation intensity for the system.

Flow in the phase space. Usually, the flow of a physical quantity is the amount of this quantity transferred in unit time through any area in the space. For the coherently accelerating systems, whose properties are identical over the whole volume, the significant parameter is the amount of such a quantity transferred in unit time through an area in the energetic or momentum space irrespective of the coordinates. The flow in the phase space (like the coherent acceleration) is related to the degree of deviation of a state of the system from the equilibrium one corresponding to the zero flow (or, what is the same, to the zero coherent acceleration).

Fractal objects — objects possessing the properties of self-similarity or scaling invariance.

Ill-posed problem — problem, whose solution is unstable with respect to the initial data or to a perturbation of the operator.

Information. It is intuitively assumed in the Shannon theory that information has content. Information decreases the total uncertainty and the informational entropy. The amount of information can be measured. However, Shannon warned as for the mechanical transfer of notions from its theory to other fields of science: “The search for ways of applying the theory of information to other regions of science ios not reduced to the trivial transfer of terms. This search can be realized in the long-term advancing of new hypotheses and their experimental verification.”

Instability by Lyapunov — instability with respect to the initial data, which leads to the exponential divergence of earlier close trajectories.

Lyapunov indices — increments of the instability with respect to the initial data (instability by Lyapunov).

Mass. Mass defect. Mass is mainly determined by the binding energy of a system. The mass defect is a change of the mass as a result of the change of the structure of the system and its constraints. For example:

- Mass of nucleons is determined by the binding energy of quarks;
- Mass of nuclei is determined by the binding energy of nucleons;
- Mass of a "shell" is determined by the binding energy of electrons, nucleons, and nuclei;
- Mass of atoms is determined by the binding energy of nuclei and electrons.

Mass force — force acting identically on all elements of a system and creating, in this case, the coherent acceleration of the system.

The example is the gravitational force acting on all particles proportionally to their masses. It is usually considered that the mass force is the reason for the appearance of a flow in the configurational space of the system.

However, in many cases where the mass force acts identically on all elements of a subsystem (separated from the whole system in some way), such a subsystem, being homogeneous in the configurational space, accelerates, i.e., a flow appears in the momentum subspace of the phase space.

The example of such a situation is given by the subsystem of electrons of a plasma in an electric field, whose intensity is more than some critical value (the runaway threshold). In this case, the plasma passes in a state with electrons running away, i.e., all electrons are coherently accelerated, and the electric field acting on the plasma plays the role of a *dominating perturbation*, which acts on the plasma and transfers the subsystem of electrons in a coherent state.

If a flow in the phase space of the system (or coherent acceleration) is not constant and is in the state with positive feedback, *the blow-up mode* arises.

Nonlocality — main characteristic of a system, being in the mode of coherent acceleration (the blow-up mode). In this case, the state of the system cannot be set by the expansion in a vicinity of the given point in infinitely small values and, hence, by the acceleration of a single order. The system is characterized by the accelerations of all orders. The property of nonlocality is characteristic of the systems in the blow-up mode, systems near a phase transition, and *MQO*.

Phase portrait — possible states of a system in its phase space; the set of trajectories of the system in its phase space.

Phase space (space of states) — multidimensional space, whose coordinates serve as parameters completely describing a state of the system.

Reference systems:

- *Inertial reference system* — reference system, in which the bodies not subjected to the action of forces move along straight lines.
- *Noninertial reference system* — reference system moving with acceleration relative to an inertial reference system.

Regularization. Operator of regularization. To obtain a stable solution of an *ill-posed problem, it is necessary* to use some special methods called the methods of regularization. It is possible to define the spaces, where the solutions of a problem become proper or, by applying *the operators of regularization* (the operators of special averaging), to change the operators defining the problem or to introduce new *observable* variables.

Resonance excitation — correspondence of the spatial and temporal structures of an external action to the internal structures of an open nonlinear system.

Self-organization — process of spontaneous ordering, formation, and evolution of structures in open nonlinear systems.

Space-time curvature — physical effect revealing itself in a deviation of geodesic lines, i.e., in the divergence or convergence of the trajectories of freely moving bodies launched from close points of the space-time. The space-time curvature is characterized by the Riemann curvature tensor.

Strange attractor — set in the phase space attracting the trajectories to itself. A strange attractor has fractal structure.

Structure — set of elements of a system with a set of stable constraints between elements:

- *Dissipative structure* — stable state of an open system, which arises as a result of the dissipation of the energy continuously supplied from outside. Prigogine developed the theory of dissipative structures to explain the behavior of systems, being far from the equilibrium. In this case, the properties of the system in small regions of the space are described by locally equilibrium functions with the values of macroscopic parameters strongly different from equilibrium ones. The strong deviation from the equilibrium in dissipative structures means large spatial gradients of macroscopic parameters of a locally equilibrium system. In this case, the moving forces of the evolution are the gradients of physical quantities.
- *Nonlocal structure.* Structure, which arises as a result of the process of self-organization, i.e., the evolution of constraints in the system in its whole spatial volume, and differs from the equilibrium system even locally. The self-organization of the system is initiated by mass forces leading to the coherent acceleration (in the absence of significant gradients of macroscopic parameters inside the system). The reconstruction of constraints and their energies in nonlocal structures occurs namely due to the coherent acceleration at the dissipated energy and the gradients inside the system tending to zero.

Synthesis — process of formation of new structures, i.e., the process of formation of new constraints.

System:

- *Open system.* System, which exchanges with the environment by energy, mass, and information.
- *Closed system.* System, which does not exchange with the environment by energy, mass, and information. Energy and information in the closed systems are conserved.

Variational principles of the evolution of a system:

- *The Hamilton principle of least action (the variational principle of the dynamics of closed systems)*
- *The Gauss principle of least compulsion (the general variational principle of the dynamics including the dynamics of the systems with constraints).*

By the principle of least compulsion, the system with ideal constraints chooses the motion with the minimal "compulsion" Z among all motions admitted by constraints, which start from the given position with given initial velocities. The free material point with mass m under the action of a given force Fon itwill have the acceleration equal to $F/m.$ If some constraints are imposed on the point, then its acceleration under the action of the same force F will be equal to a different value w. The deviation of the motion of the point from free motion due to the action of a constraint will depend on the difference of these accelerations $F/m - w$. The quantity Z proportional to the square of this difference is called "compulsion". For a single point, $Z = \frac{1}{2}m(F/m - w)^2$

- *Hertz least-curvature principle (the variational dynamical principle, which is the closest to the Gauss principle and the most convenient for the systems with constraints).* From all trajectories admissible by constraints, the trajectory with the least curvature will be realized. This principle is also called the principle of straightest path and is closely related to the principle of least compulsion, because the quantity called the "compulsion" is proportional to the square of the curvature. For ideal constraints, both principles have the same mathematical representation.
- *Principle of minimal entropy production (Prigogine principle of evolution for dissipative systems and structures).* In 1947, I. Prigogine introduced the notions of entropy production and entropy flow, gave the so-called local formulation of the second origin of thermodynamics, and proposed the principle of local equilibrium. He showed that, in the stationary state, the entropy production rate in a thermodynamical system is minimal (Prigogine theorem), and the entropy production flor irreversible processes in an open system tends to a minimum (Prigogine criterion).
- *Principle of dynamical harmonization (the most general principle of dynamical evolution of systems with varying constraints)*

Appendix 2. Basic notation

η — order parameter
D_f — fractal dimension
q — parameter of nonequilibrium
q_s — parameter of similarity
α_d — parameter of domination
Q_{imp} — parameter of impactness
τ_{eff} — effective duration of the operation of a driver
τ_{dis} — effective duration of the dissipation
a_{cog} — coherent acceleration
F_m — mass force
F_{str} — mass force initiating the formation of a structure
σ_S — entropy production
J — action
S — entropy
u_T, p_T — thermal velocity and momentum

m_{str} — structure inertia (structure mass)
m — mass
B, sB — binding energy and specific binding energy per nucleon
L_{str} — Lagrange function at the formation of a structure
Z_{dh} — dynamical harmonization functional
$\vec{A}$, φ — vector and electrostatic potentials
A, Z — mass and charge of a nucleus
l_+ — delocalization scale
l_- — – — localization scale
δ — deformation
$\delta(q_s)$ — damping decrement or increment of the instability
D_{q_s} — Jackson operator with the parameter of similarity q_s
Q — quality of an oscillatory circuit
κ — space–time curvature
g_{ik} — space–time metric
R_{ik} — Riemann curvature tensor

Appendix 3. Main relations for the Jackson operators (integro-differential operators of quantum analysis)

The fractal media are characterized by the properties of the similarity of basic quantities at a variation of the space scales. Therefore, The most natural generalization of the notion of derivative is the Jackson derivative [4], in which the scaling operation (with the coefficient of similarity q_s) is used for the determination of the rate of a process instead of the operators of shift:

$$D_{q_s} f(x) = \frac{f(q_s x) - f(x)}{q_s x - x}. \quad (1)$$

In the limiting case, the Jackson derivative passes to the ordinary one: $Df(x) = \lim_{q_s \to 1} D_{q_s} f(x)$.

The question arises: Which functions are the eigenfunctions of the operators of Jackson q-derivatives? On the basis of the development of the notion of q-derivatives, the so-called quantum analysis was constructed, in the frame of which the generalizations of many significant mathematical relations were found. For example, let us calculate the quantum q-derivative of a power function:

$$D_{q_s} x^n = \frac{(q_s x)^n - x^n}{(q_s - 1)x} = \frac{{q_s}^n - 1}{q_s - 1} x^{n-1} = [n]_{q_s} x^{n-1}, \quad (2)$$

where $[\alpha]_{q_s} = \frac{{q_s}^\alpha - 1}{q_s - 1}$ is the Jackson q-number, whose limits are $\lim_{q_s \to 1} [\alpha]_{q_s} = \alpha$ and $\lim_{q_s \to \infty} [\alpha]_{q_s} = q_s^{\alpha-1}$. It is simple to calculate the derivative of a function possessing the property of similarity. Let $f(q_s x) = q_s^\alpha f(x)$, then

$$D_{q_s} f(x) = \frac{({q_s}^\alpha f(x)) - f(x)}{(q_s - 1)x} = \frac{{q_s}^\alpha - 1}{q_s - 1} \frac{f(x)}{x} = [\alpha]_{q_s} \frac{f(x)}{x}. \quad (3)$$

The eigenfunction of the Riemann derivative is the exponential function e^x, which can be expanded in a power series $e^x = \sum\limits_{k=0}^{\infty} \frac{x^k}{k!}$, where $k!$ is a factorial. Quantum analysis uses widely the q-generalization of the exponential function e_q^x, whose power expansion contains the generalization of $k!$, which is replaced by $[k]_{q_s}!$:

$$[k]_{q_s}! = \begin{cases} 1, & k = 0 \\ [k]_{q_s} [k-1]_{q_s} \ldots [1]_{q_s}, & k \geqslant 1 \end{cases}. \tag{4}$$

In other words, the power series for the q-exponential function takes the form

$$e_{q_s}^x = \sum_{k=0}^{\infty} \frac{x^n}{[k]_{q_s}!}. \tag{5}$$

It is easy to see that such a definition implies that the function e_q^x is the eigenfunction of the operator D_q:

$$\begin{aligned} D_q e_{q_s}^x &= D_{q_s} \left(\sum_{k=0}^{\infty} \frac{x^k}{[k]_{q_s}!} \right) = \sum_{k=0}^{\infty} \frac{1}{[k]_{q_s}!} D_{q_s} \left(x^k \right) \\ &= \sum_{k=1}^{\infty} \frac{[k]_{q_s}}{[k]_{q_s}!} x^{k-1} = \sum_{k=1}^{\infty} \frac{1}{[k-1]_{q_s}!} x^{k-1} = e_{q_s}^x. \end{aligned} \tag{6}$$

The quantum derivative is a linear operator. Therefore, the q-derivative of a linear combination of functions can be presented in terms of the derivatives of separate functions by the ordinary relation. However, the q-derivative of a product of functions has already some specific features.

Definition (1) yields the relations for the derivatives of a product of functions that differ from ordinary relations by the absence of symmetry. Namely, two different relations are simultaneously valid for the derivative of a product of functions:

$$\begin{aligned} D_{q_s} \left(f(x) g(x) \right) &= f(q_s x) D_{q_s} g(x) + g(x) D_{q_s} f(x), \\ D_{q_s} \left(f(x) g(x) \right) &= f(x) D_{q_s} g(x) + g(q_s x) D_{q_s} f(x). \end{aligned} \tag{7}$$

For the functions possessing the similarity, $f(q_s x) = {q_s}^{\alpha} f(x)$ and $g(q_s x) = {q_s}^{\beta} g(x)$, we obtain

$$\begin{aligned} D_{q_s} \left(f(x) g(x) \right) &= f(x) D_{q_s} g(x) + {q_s}^{\beta} g(x) D_{q_s} f(x) \\ &= f(x) D_{q_s} g(x) + g(x) D_{q_s} f(x) + \left({q_s}^{\beta} - 1 \right) g(x) D_{q_s} f(x). \end{aligned} \tag{8}$$

Hence, the parameter of similarity q_s of the quantum differentiation characterizes simultaneously the degree of its asymmetry.

In addition, quantum analysis considers the operators, which are inverse to the derivatives — the operators of q-primitives. The function $F(x)$ is called the q-primitive for a function $f(x)$, if $D_q F(x) = f(x)$, and is denoted by $\int f(x) d_q x$. It is easy to see that if a function $f(x)$ is set by a power series $f(x) = \sum\limits_{k=0}^{\infty} a_k x^k$, then

$$\int f(x) d_{q_s} x = \sum_{k=0}^{\infty} \frac{a_k}{[k+1]_{q_s}} x^{k+1} + C. \tag{9}$$

Sometimes, it is convenient to use the formal definition of the Jackson integral for the q-primitive of a function $f(x)$:

$$\int f(x) d_{q_s} x = (1 - q_s)\, x \sum_{k=0}^{\infty} {q_s}^k f\left({q_s}^k x\right). \tag{10}$$

We note that the Jackson q-numbers $[x]_{q_s}$, which are expressed in terms of the parameter of similarity q_s, are closely related to the Tsallis nonextensive entropy for the states with the parameter of nonequilibrium q:

$$S_q = -\sum_i p_i^q \ln_q(p_i) = \frac{1 - \sum_i p_i^q}{q - 1}.$$

In the definition of entropy, we apply the generalized logarithm

$$\ln_q x = \frac{x^{q-1} - 1}{q - 1},$$

which satisfies the relation

$$\ln_q(xy) = \ln_q(x) + \ln_q(y) + (1 - q)\ln_q(x)\ln_q(y).$$

The main property of the generalized entropy S_q consists in that it is not already the extensive function. If the system is divided into two independent subsystems A and B, then

$$S_q(A + B) = S_q(A) + S_q(B) + (1 - q) S_q(A) S_q(B). \tag{11}$$

Deviations from the symmetry and the ideality in this relation are determined, like that in (8), by the deviation of the relevant parameter from 1.

The majority of equilibrium physical parameters of closed ideal systems are expressed via ordinary exponential functions coinciding with their generalized analogs for the coefficient of nonequilibrium $q \approx 1$ and the coefficient of similarity $q_s \approx 1$. The degree of deviation from the thermodynamic equilibrium and the ideality is determined by the deviation of the mentioned parameters from 1. The nonideal states of the system must manifest themselves, naturally, in the oscillatory processes, which are realized in fractal media.

A model of oscillatory processes in fractal media on the basis of quantum analysis

To study the peculiarities of oscillatory processes in fractal media, we consider the generalization of the trigonometric functions on the basis of q-exponential functions (5). In the frame of quantum analysis, the following new functions are introduced:

$$\cos_q(z) = \frac{e_q^{iz} + e_q^{-iz}}{2}; \qquad \sin_q(z) = \frac{e_q^{iz} - e_q^{-iz}}{2i}. \tag{12}$$

Using relations (6) for the quantum derivative of a generalized exponential function, it is easy to obtain that the functions introduced with the help of relations (12) satisfy the relations similar to the relations for trigonometric functions:

$$D_q \cos_q(z) = -\sin_q(z), \qquad D_q \sin_q(z) = \cos_q(z). \tag{13}$$

These q-functions pass into the ordinary trigonometric functions as $q \to 1$. However, the deviation of the former from the latter increases with the deviation of the parameter of nonextensity q from 1. In Fig. 1, we present the plots for the q-trigonometric functions $\sin_q(t)$ for various parameters of similarity.

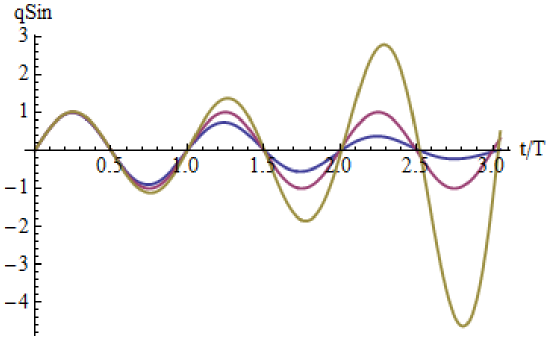

Fig. 1. Simplest oscillatory process in a fractal medium. Plots of the functions $\sin(t)$ and $\sin_q(t)$ are given for the parameters of similarity q_s=0.95 and q_s=1.05

As is seen from Fig. 1, the oscillatory process described by q-trigonometric functions has character of a dissipative process. Let us analyze this analogy in more details. Consider the simplest self-similar oscillatory process, which is described by the simple equation for a fractal oscillator with the use of the Jackson derivatives:

$$D_q(D_q f(x)) + \omega^2 f(x) = 0. \tag{14}$$

By the direct substitution, it is easy to verify that the general solution of this equation is the function $f(x) = C_1 \sin_q(\omega x) + C_2 \cos_q(\omega x)$. The case shown in Fig. 1 corresponds to the initial conditions $f(0) = 0$ and $D_q f(0) = 1$, for which $f(x) = \sin_q(\omega x)$.

In practical applications, it is convenient to approximate the Jackson q-functions, which are represented by infinite series, by their finite algebraic expressions. It is natural to make it with the use of nonequilibrium quasipower generalizations of the exponential function, $\exp_q(x) = (1 + (1-q)x)^{\frac{1}{1-q}}$, which allow us to write the quasipower generalizations of the trigonometric functions:

$$qCos(z) = \frac{\exp_q(iz) + \exp_q(-iz)}{2}; \quad qSin(z) = \frac{\exp_q(iz) - \exp_q(-iz)}{2i}. \tag{15}$$

We now consider the generalized exponential functions $\exp_q(-z_k)$ and $e_{q_s}^{-z_k}$ on the interval $0 \leqslant z_k \leqslant 4$ and define the connection between the parameter of nonequilibrium q and the parameter of similarity q_s from the condition of minimum for $\sum_{k=1}^{N} \left(\exp_q(-z_k) - e_{q_z}^{-z_k}\right)^2$. As a result, we obtain

$$q(q_s) = \begin{cases} 2.023 - 1.5608 q_s + 0.5380 q_s^2, & q_s \leqslant 1 \\ 1.7005 - 0.9234 q_s + 0.2223 q_s^2, & q_s > 1 \end{cases}. \tag{16}$$

In Fig. 2 on the left, we show the self-similar oscillatory process $f(x) = \sin_q(\omega x)$ and its approximation with the generalized exponential functions $\exp_q(-z_k)$, for which the parameter q is determined by relation (16). We indicate a sufficiently high accuracy of the approximation. On the right, we present the phase portrait of this self-similar oscillation.

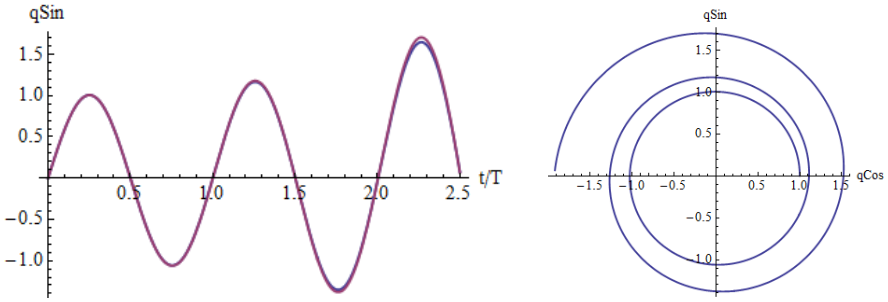

Fig. 2. Simplest oscillatory self-similar process, its approximation wlith generalized exponential functions (on the left), and the corresponding phase trajectory (on the right).

It is seen that the oscillatory process with the coefficient of similarity q_s corresponds approximately to the unstable oscillatory process

$$g(x) = e^{-\delta x} \sin(\omega x + \Delta\varphi)$$

described by the equation with ordinary derivatives for an oscillator with negative damping δ:

$$D_x^1\left(D_x^1 g(x)\right) - \delta D_x^1 g(x) + \omega^2 g(x) = 0. \tag{17}$$

The parameters of similarity and damping are connected by a relation that will be determined from the condition of the maximal coincidence of the phase trajectories of a self-similar oscillation and unstable (or decaying) linear oscillations by the method of least squares:

$$\delta(q_s) = \begin{cases} 3.4931(1-q_s)^{0.6473}, & q_s < 1 \\ -10.8126(q_s - 1)^{0.7969}, & q_s > 1 \end{cases}. \tag{18}$$

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Метафізика духовної впливовості Київської духовної академії (на прикладі сім'ї Булгакових)

Г. І. Волинка[1], *В. О. Дорошкевич*[1], *Н. Г. Мозгова*[1]

Анотація. Сім'я Булгакових суттєво вплинула на розвиток світової культури, вона являється частиною культурної спадщини Києва і нові знання про творчість представників цієї сім'ї являють собою величезний історичний та науковий інтерес, а також доповнюють наші знання про епоху релігійно-філософського ренесансу кінця XIX — початку XX століття. Великий вплив на формування світогляду Афанасія Івановича Булгакова (батька Михайла Булгакова) і Миколи Івановича Булгакова (дядька знаменитого письменника) мала Київська духовна академія. У даній публікації особлива увага приділяється дослідженню маловивченої біографії Миколи Івановича Булгакова, випускника КДА, викладача Тифліської православної семінарії, в якій навчався Йосип Сталін. Події, які відбулися в цій семінарії та роль в цих подіях М. І. Булгакова, знайшли своє відображення в останній п'єсі Михайла Булгакова «Батум».

Авторитетність навчального закладу — річ вкрай небайдужа для кожного, хто з ним неформально і щиро пов'язаний. Але річ ця є одночасно надто невизначеною, майже невловимою. Про неї можна говорити все що завгодно, оскільки вона не піддається об'єктивним вимірам, спостереженням, фіксаціям. Вона немовби ховається за видимими реаліями, лише натякаючи на свою наявність та метафізичну природу. Єдине, де вона себе являє з переконливою автентичністю, — це факти професійних, наукових, моральних перемог вихованців навчального закладу. Саме через успіхи своїх вихованців навчальний заклад неначе привідчиняє метафізику свого духовного потенціалу і авторитету, транслює його іншим людям, впливаючи зрештою на ціле суспільство. Метафізичний потенціал його духовної впливовості здійснюється через вербальну і позавербальну міжособистісну комунікацію, опосередковану смислами опредмеченого успіху, котрий дану впливовість засвідчує.

Цю тезу раптом і по-новому висвітив один разючий факт. Ні, він не зводиться до якоїсь нової інформації про суперечливі взаємини Й. В. Сталіна і відомого російського письменника М. А. Булгакова. По-перше, їх стосунки є досить відомими для кожної освіченої людини. По-друге, предметом нашої розвідки є Київська духовна академія, а з нею письменник М. А. Булгаков був пов'язаний лише опосередковано, хоча. . . Факт, про який йтиметься нижче, — доля зовсім іншої людини, — рідного дядька

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письменника, безпосереднього вихованця академії Миколи Івановича Булгакова, особистість якого залишається поки-що невідомою навіть маститим булгаковознавцям. Для прикладу: видана у Москві у 1998 році «Булгаківська енциклопедія» Б. Соколова ні словом не згадує про нього [32].

Між тим, М. І. Булгаков — особа явно непересічна, оскільки саме йому, після закінчення Київської духовної академії (далі — КДА) у 1891 р. і отримавши направлення на викладацьку діяльність в Тифліську православну семінарію, довелося мати справу з генетичними релігійно-сектанськими коренями і першими паростками сталінізму, прихованими пізніше в ортодоксальній історії ВКП(б). Саме він одним з перших рішуче протистояв їм, безпосередньо і опосередковано вплинувши на долю революційних соратників юного Сталіна та на самого вождя. У книзі про одного з них (Володимира Кецховелі — *авт.*), Л. П. Берія згадує про «якогось викладача Булгакова», називаючи його самодуром і людиноненависником [3; с. 8]. Про особу М. І. Булгакова добре знав і Сталін; письменник М. А. Булгаков здогадувався про це. Цим, напевно, він і пояснював суперечливо-неоднозначне ставлення до себе з боку Сталіна. Недаремно ж свою останню п'єсу «Батум» письменник присвятив юності вождя і, сподіваємось, своєму дядькові — Миколі Івановичу.

Про особу М. І. Булгакова та суттєві подробиці його перебування в Тифлісі ми довідались спочатку з рукописів [28] (їх надав одному з авторів даної статті Київський Музей Однієї (Андріївської) Вулиці), а згодом — з частково опублікованих О. та Д. Шльонськими спогадів його однокурсника Володимира Петровича Рибинського (1867–1944) — одного з останніх професорів та інспекторів КДА [29]. Професор Рибинський пише про Булгакова, що Микола Іванович займав у Тифліській православній семінарії посаду помічника інспектора і надто вимогливо та жорстко ставився до її вихованців. При цьому В. П. Рибинський посилається на свідчення іншого свого однокурсника — архімандрита Іоаннікія (Івана Олександровича Єфремова), котрий працював у Тифлісі разом з М. І. Булгаковим на посаді інспектора семінарії [29, с. 178], а з вересня 1893 р. отримав високу посаду ректора Київської духовної семінарії. [14, с. 15]. Напевно після повернення з Тифлісу Єфремов і розповів Рибинському про Булгакова, який завдав йому багато клопоту своєю нетерпимою прискіпливістю. «Приведе, бувало, якого-небудь грузина (семінариста — *авт.*) до мене і каже: ось я привів вам цього [впертого ледацюгу], робіть з ним, що хочете. — У того очі горять, то ж чекай, що вийде неприємність» [28, с. 145].

Згадуючи своїх найближчих друзів по Академії, Рибинський, посилаючись крім Єфремова і на інших однокурсників, підкреслював, що в Тифлісі М. Булгаков дуже змінився: «На студентській лаві це була добродушна, весела, щедра людина, яка потім виявилась нудною, сухою і практичною. Зустрічі наші більше не мали справжнього товариського характеру» [29, с. 178].

Що ж трапилось з Миколою Івановичем в Тифліській православній семінарії? Невже він зазнав якогось духовного зламу? Звідки і чому з'явилась жорсткість, сухість і надмірна вимогливість, про яку В. П. Рибинському сповіщає Іоаннікій (І. О. Єфремов)? І чому в тому ж Тифлісі не зазнав подібних метаморфоз останній (Єфремов — *авт.*), за словами В. П. Рибинського психічно неврівноважений, внаслідок перенесеної на III курсі хвороби, чоловік, який пізніше вірогідно вдається до самогубства [28, с. 132]? А може В. П. Рибинський не враховує якихось важливих обставин, пов'язаних з перебуванням М. І. Булгакова в Тифлісі, обставин, дії яких уник Іоаннікій (І. О. Єфремов), залишивши Грузію до вересня 1893 р. [14, с. 15]? Зрештою, можливо ці гіпотетичні обставини зовсім не зламали вдачі Миколи Івановича, а просто зробили його більш серйозним, відповідальним, здатним до духовного подвижництва і послідовного відстоювання закладених в КДА принципів професійної честі та православного віросповідання?

Щоб якось прояснити вищезазначені питання, безпосередньо пов'язані з духовними впливами КДА через своїх вихованців, ми почали розшукувати більш змістовну інформацію щодо обставин перебування М. І. Булгакова у Тифлісі. Першим кинулось у вічі побіжне твердження О. та Д. Шльонських, висловлене в примітках до публікації спогадів професора В. П. Рибинського. Воно було для нас дуже цікавим, бо зміст їх здогадки безпосередньо стосувався предмета нашої власної зацікавленості. Вони писали, «...що саме під час викладання М. І. Булгакова в Тифліській духовній семінарії навчався Й. Джугашвілі (Сталін), якого було вигнано звідти 1899 р.» [29, с. 180]. Якби це твердження було підкріплене хоча б хронологічними збігами, воно б, без сумніву, стало помітним внеском у булгакововознавство.

Але відомо, що Сталін вступив до Тифліської семінарії у вересні 1894 р. Коли ж покинув Грузію М. І. Булгаков, принаймні для нас, лишається невідомим й сьогодні. У нашій публікації, підготовленій у 2000 році [21], ми відобразили суттєві факти перебування М. І. Булгакова в Тифлісі кінця 1893 р. Ми вперше опублікували у тексті нашої статті його лист з Тифлісу до Києва М. Петрову, датований 16 квітня 1894 р. Тобто можна з впевненістю стверджувати, що він перебував у Тифлісі протягом січневого і квітневого триместрів 1894 р. Проте чи знаходився він там після вересня? Дане питання не має однозначної відповіді. Ясності не додає навіть та обставина, що в останньому (грудневому) номері 1894 р. «Духовного вісника Грузинського Екзархату» розміщена стаття М. І. Булгакова про баптизм. Треба шукати точну дату його від'їзду з Тифлісу.

Проте, якщо навіть життєві шляхи Й. В. Сталіна і М. І. Булгакова не перетинались у суто фізичному часопросторі, вони, без сумніву, перетнулись у хронотопі метафізичних реалій. Вождь не міг не знати про непохитно послідовного, жорсткого і вимогливого викладача-вихователя Булгакова. В юності він напевно довідався про нього та його круту вдачу від своїх колег-семінаристів, які неодноразово у своїх заявах Екзарху Грузії вимагали його звільнення [17, с. 174], у зрілому ж віці йому не могли не нагадати цього вже цитовані нами документи, підготовлені Л. П. Берією у 1937 р. і присвячені революційній діяльності Ладо Кецховелі [3] — соратника Сталіна і друга з дитинства, ще з часів навчання обох у Горійському духовному училищі, — який вступив до семінарії у тому ж 1891 р., коли там з'явився випускник КДА М. І. Булгаков. Про відносини останнього з Ладо Кецховелі, та про пов'язані з цим події розповімо трохи пізніше.

Виходячи з припущення, що остання п'єса письменника М. А. Булгакова, присвячена не лише Сталіну, а насамперед його дядькові, ми вирішили звернутися до неї, сподіваючись знайти в документах, з якими працював М. А. Булгаков, інформацію про обставини перебування в Грузії його рідного дядька. Не думаємо, щоб звернувшись до «Батуму», ми далеко відійшли від головної теми нашого дослідження — часопростору духовної впливовості Київської академії, її впливів на перебіг історичних подій, на людські долі, на долю Булгакова — письменника.

«Батум» — остання п'єса М. А. Булгакова, яка була написана протягом першої половини 1939 р. Щодо мотивів написання твору існує багато суперечок і кардинально протилежних думок, до яких дав привід сам автор. Ще у 1930 р. в листі до «Уряду СРСР» від 28 березня він писав: «Після того, як усі мої твори були заборонені, серед багатьох громадян, яким я відомий як письменник, почали лунати голоси, які радять мені одне і теж: «Створити комуністичну п'єсу», а крім того, звернутись до Уряду СРСР з листом розкаяння, що містить в собі відмову від моїх поглядів, висловлених мною у літературних творах, і переконання у тому, що відтепер я буду працювати як прихильник щодо ідеї комунізму, як письменник — попутник. ... Цієї поради я не дотримався ... Спроб створити комуністичну п'єсу я навіть не намагався робити, знаючи наперед, що така п'єса у мене не вийде» [27, с. 443–444]. Треба зазначити, що М. А. Булгаков дійсно відмовляв проханням вдатися до комуністичної тематики, зокрема, відхилив пропозицію написати п'єсу до 20-ї річниці Жовтневої революції [див.: 16, с. 418]. І раптом починає активно працювати над п'єсою про початок революційної діяльності Сталіна...

Прозора розбіжність між декларуваннями і дійсними справами дала привід деяким булгаковознавцям засумніватись у щирості драматурга, у послідовності його поведінки. Так, у книзі В. Г. Боборикіна говориться, що написанням «Батуму» М. А. Булгаков пішов «проти своєї совісті» [4, с. 198]. В інших стверджується, що він написав «вірнопідданську п'єсу», у якій від «булгаковської майстерності не залишилось нічого» [30, с. 58]. Дружина ж письменника Олена Сергіївна Булгакова, на очах якої створювався «Батум», однозначно висловила своє ставлення до твору під час зустрічі з М. О. Чудаковою у 1969 р.: «Я страшенно люблю цю п'єсу» [35, с. 204]. Створив її справжній і безкомпромісний Майстер — «безстрашний — завжди і в усьому ... Втілена совість. Непідкупна честь» [12, с. 282].

Якщо взяти до уваги вищенаведені твердження близьких сучасників Майстра, виходить, що розбіжності між його заявами і справами немає. Але тоді «Батум» не є «комуністичною п'єсою», яка прославляє Сталіна. Цей принциповий висновок опосередковано підтверджується у роздумах М. Петровського [див.: 25] А. Нінова [див.: 23], О. Смелянського [див.: 30]. Справді, якщо уважно прочитати «Батум» і взяти до уваги, скажімо, промову Ректора у першому акті п'єси при виключенні Й. В. Джугашвілі з семінарії (1899 р.), важко не побачити, що вона принципово звинувачує його як злочинця, котрий «сіє зле сем'я у нашій країні» [7, с. 230]. «Похмура і по-своєму сильна промова Ректора семінарії, який накликає кару Господню на голову молодого відступника, була, — зазначає А. Нінов, — напевно, єдиним у своєму роді звинуваченням Сталіна в радянській драматургії кінця 30-х років, яке відмовляло ідеї безбожжя та політичного бунту проти існуючої влади у будь-якому моральному виправданні» [23, с. 46].

Виходить Сталін для М. А. Булгакова виступає своєрідним антигероєм, кримінальним персонажем, котрий міг би бути цікавим Булгакову лише як сатирику, якби про нього можна було писати сатиричний твір. То ж його звернення до теми юності вождя, окрім вже згаданих ілюзорних «вірнопідданських» мотивів, повинно мати й іншу, справжню, мотивацію. Нею, як ми вже зазначали, може бути виключно пам'ять про свого дядька — достойну, високоосвічену, принципову і щиру людину.

Майже всі булгаковознавці і до сьогодні одностайно стверджують, що при написанні «Батуму» М. А. Булгаков користувався лише одним джерелом — розкішно виданою книгою «Батумська демонстрація 1902 року» — збірником документів про цю подію, спогадами про Сталіна. Книга була видана у 1937 р. Партвидавом при ЦК ВКП(б) з передмовою Л. П. Берії [2]. Майстер опрацював цю книгу дуже ретельно, про що свідчать чисельні помітки в екземплярі, котрий належав йому. Допоміжним джерелом при роботі над «Батумом» була апологетична біографія Сталіна, яку написав Анрі Барбюс і яка вийшла у 1936 р. російською мовою. [1].

Необхідно також підкреслити, що вищеназваний збірник, надрукований в офіційних органах, містить викривлення історичної дійсності. Сталін насправді не був керівником Батумської демонстрації. Про це можна дізнатись із збереженої в партійних архівах «Доповіді Батумського комітету РСДРП» II з'їзду РСДРП, який відбувся у 1903 р. В ньому описується діяльність комітету за період 1900–1902 рр. і його вплив на Батумський страйк робітників у 1902 р., який був організований, як сказано в доповіді, передовими робітниками. Керівниками ж комітету насправді були соціал-демократи І. Рамішвілі та Н. Чхеїдзе, які жили в Батумі із середини 1890-х рр. [див.: 15]. Правим виявився американський дослідник біографії Сталіна Р. Такер, який писав про Батумський страйк: «Чи грав яку-небудь роль Джугашвілі у цих подіях лишається невідомим» [33, с. 94]. Саме тому можна визначити другу й наступні картини «Батуму» як лише художній опис офіційно представленої історії (хоча й майстерний), бо автор повинен був строго додержуватись офіційної версії, всупереч творчим намірам. Чи здогадувався М. А. Булгаков про те, що офіційні матеріали фальсифікують дійсну картину подій? Думається, що так. На це посередньо може вказувати природа образу Сталіна, зображеного в першій сцені п'єси. Свідченням їх брехливості для письменника було й те, що ювілейний збірник «Батумська демонстрація 1902 року» за редакцією Л. П. Берії з благословення Сталіна був блискуче виданий, а п'єса Булгакова про ті ж події категорично заборонена.

Знаючи про ґрунтовно-прискіпливе ставлення Майстра до першоджерел, на яких базувались його попередні твори, ми продовжували посильні спроби розшукати хоч якісь натяки на існування додаткових витоків «Батуму». На радість це вдалося. У статті А. Нінова, який мав змогу працювати з першою чорновою редакцією п'єси та допоміжними матеріалами, ми читаємо: «Серед реальних джерел для першої картини в записах Булгакова названі: «Духовний вісник Грузинського Екзархату» за 1894–1897 рр. Його особливу увагу привернули №№ .1, 23 і 24 за 1894 р. і № 24 за 1897 р. ...Булгакову була відома також стаття «Зі спогадів російського вчителя грузинської Православної Духовної семінарії в Тифлісі» (1907) — згадка про неї збереглася в чорнових записах до п'єси» [23, с. 45–46].

Було зроблене припущення, що в названих номерах «Духовного вісника...» мають міститись матеріали не про Сталіна (з вересня 1894 р. він став учнем семінарії і у «Віснику» за 1894 р. він міг фігурувати лише у загальному списку першокурсників), а більш цікава для письменника інформація, щось про його дядька. Ми додали: якби М. А. Булгаков переважно цікавився лише долею вождя, він звернувся б до номерів за 1899–1900 рр., бо саме в них мали міститись відомості про обставини його виключення з семінарії, такі важливі для написання першої картини. Але їх немає серед згаданих А. Ніновим реальних джерел до неї ... Хто ж тоді насправді цікавив М. А. Булгакова? Звичайно, що дядько, і лише він.

Хоча нам і не вдалось поки-що знайти № 24 за 1897 р., та згадану вище статтю, але всі 24 номери за 1894 р., до речі, з додатками до офіційних частин Вісника, ми розшукали. І у всіх відмічених письменником №№ 1, 23 і 24 (спарений номер) є лише небайдужі для нього особисто відомості про М. І. Булгакова і навіть дядькова промова в Тифліській семінарії про ворожість державі баптизму. В номерах, які, начебто, обминула увага Майстра, (№. 5, 10, 15) ми знайшли неабияку інформацію про заворушення учнів Тифліської семінарії 1–4 грудня 1893 року.

Вже на перших сторінках «Духовного вісника...» № 15 знаходиться коротке повідомлення про заколот учнів [34, с. 1] під керівництвом М. Цхакая і В. Кецховелі [14, с. 7], про рішення Св. Синоду закрити семінарію до 1 вересня 1894 року, про виключення 87 її вихованців (у списку фігурує і вже згадуваний Ладо Кецховелі) без права поновлення в семінаріях Росії [34, с. 1–3]. Правда, через півроку Св. Синод пом'якшив попереднє рішення і раніше виключені семінаристи, отримали дозвіл на продовження навчання в інших, крім Тифліської, семінаріях Київського духовно-навчального округу.

Щоби читач мав уявлення про Київський духовно-навчальний округ, очолюваний КДА, наведемо деяку інформацію про неї, а потім повернемось до нашого сюжету.

Існувала Київська духовна академія (спадкоємиця Могилянки) відносно недовго, з 1819 по 1920 р., тобто трохи більше ста років. У ці часи простір її адміністративно-територіальної влади періодично змінювався, але в цілому був надзвичайно великим. Так, за академічним Статутом 1808–1814 рр. до Київського духовно-навчального округу, на чолі якого з 1819 р. стояла КДА, окрім Київської єпархії у неї входили Чернігівська, Волинська, Подільська, Полтавська, Катеринославська, Харківська, Херсонська, Таврійська, Донська, Воронезька, Курська, Орловська, Мінська, Варшавська і Кишинівська. З 1835 р. до складу округу входять також Грузинська та Імеретинська єпархії. Єпархіальні середні духовно-навчальні заклади — семінарії (в кожній єпархії існувала лише одна семінарія) були безпосередньо підпорядковані КДА в усіх галузях їх життєдіяльності — адміністративній, господарській, навчальній, науковій, кадровій тощо. Духовне, наукове, економічне життя початкових навчальних закладів в єпархіях — духовних училищах — також було підпорядковане академії, але вже опосередковано, через семінарії.

Саме до Київської семінарії звертається головний заворушник тифліських заворушень і друг Й. В. Джугашвілі Ладо Кецховелі з проханням зарахувати його до III класу з вересня 1894 р. Згадаємо, що її ректором у цей час був Іоаннікій (Єфремов), вже згадуваний однокурсник М. І. Булгакова, його колега по Тифлісу, котрий з 1892 р. у якості інспектора добре знав молодого грузинського заворушника, але дозволив йому стати семінаристом у Києві. Правда, згодом і пожалкував. . . . Проте, це вже тема іншої наукового дослідження.

Про більш детальні подробиці заворушень у Тифлісі та інші обставини перебування Ладо Кецховелі в Київській семінарії (тут він пережив таку ж духовну кризу, як і М. І. Булгаков у Тифлісі) ми дізнались з перевиданого в 1969 р. збірника документів і матеріалів «Ладо Кецховелі»[19, с. 177–178]. В ньому також йдеться про страйк семінаристів на початку грудня 1893 р., приводиться повний текст заяви учнів семінарії екзарху Грузії від 1 грудня 1893 р.,[див.: 19, с. 174–175], де вони вперто вимагають: «внаслідок неможливості виправити характер вчителя Булгакова та двох. . . . наглядачів Покровського та Іванова, вигнати їх. Вони для нас є злими ангелами, Мефістофелями, які підбурюють нашу совість і душу постійними площадними лайками і необґрунтованими інквізиторськими розслідуваннями» [19, с. 174].

Розмірковуючи над причинами страйку вихованців Тифліської православної духовної семінарії, ми не можемо повністю погодитись з її поясненнями, розміщеними у збірнику документів і матеріалів, присвячених революційній діяльності Ладо Кецховелі [19, с. 174–189]. Ми не згодні також і з тим, як зображений у цьому збірнику перебіг подій, оскільки в ньому висвітлюються лише деякі, упереджено відібрані значно пізніше, наслідки реальних, ретельно замовчуваних причин. Їх ми і спробуємо розглянути, засновуючись на ще невідомих документах.

Так, напередодні страйку, а саме 30 листопада 1893 р. семінарія святкувала своє храмове свято — день пам'яті Святого Апостола Андрія Первозваного. На святі були Екзарх Грузії, інші архієреї. В семінарській церкві служба продовжувалась всю ніч [13, с. 24–25]. Ще вдень, перед прибуттям Екзарха Грузії до семінарії, в її актовій залі відбувся Річний урочистий акт, де були присутніми всі керівники, викладачі, семінаристи, і на якому з великою, вже згадуваною промовою «Баптизм як секта, небезпечна для держави» виступив Микола Іванович Булгаков [10]. Він і раніше виступав з аналогічних питань у Тифлісі. Прикладом може слугувати віднайдена нами стаття «Порівняння чудес Ісуса Христа та його Апостолів з чудесами Старозавітними», яка являла собою оброблений для опублікування ескіз бесід з молоканами-суботниками [9]. Є підстави стверджувати, що Михайло Булгаков, працюючи над «Батумом», не знайшов цієї статті. Але безумовно він читав текст промови «Баптизм як секта. . . ». Промова, як виявилось, у багатьох смислах стала справжньою подією на святі.

Дізнавшись про цю подію, ми подумали, а чи не міг виступ М. І. Булгакова слугувати безпосереднім приводом для початку заворушення семінаристів 1 грудня 1893 р.? Хоча б тому, що його виголосив такий «неприємний» для учнів семінарії викладач філософських та релігієзнавчих дисциплін, як М. І. Булгаков. До речі, В. П. Рибинський помилився у своїх спогадах, назвавши його помічником інспектора семінарії [28, с. 145]. Можливо, він суто добровільно допомагає своєму однокурсникові архімандриту Іоаннікію (Єфремову) підтримувати дисципліну у духовному навчальному закладі. На користь сказаного свідчить й те, що після від'їзду Іоаннікія до Києва, Булгаков не став інспектором; на цю посаду був призначений ієроманах Гермоген, який обіймав її до закриття семінарії у грудні 1893 р. Можна припустити також, що в результаті заворушень на посаді інспектора закономірно став грузин. Ним цілком міг бути щойно пострижений у монахи авторитетний князь Давид Абашидзе, який прийняв ім'я Антонія і пізніше «будучи інспектором Тифліської духовної семінарії, виключав з неї студента-революціонера Й. В. Джугашвілі» [18, с. 8]. До речі, останній закінчив свій життєвий шлях у Києві 1942 р. у високому сані схимоархієпіскопа. Його могила й сьогодні знаходиться на території Ближніх печер Києво-Печерської Лаври. Можливо, що обом наступникам Єфремова, М. І. Булгаков також допомагав підтримувати належний порядок серед семінаристів. Тому саме на М. І. Булгакова і виплеснулись їх емоції під час його доповіді про баптизм. Але справа, напевно, не лише у факті виступу ненависного викладача, а й в упередженій готовності слухачів сприйняти суто теоретичну промову з певною, однозначною на неї реакцією. Спробуємо звернутись до змісту промови з врахуванням соціально-політичного і ідеологічного контексту тогочасної Грузії, як частини Російської імперії.

Кінець XIX — початок XX ст. був складним періодом у розвитку соціально-політичних і релігійно-філософських течій в Європі. Послідовники Ніцше і Володимира Соловйова, «західники» і «слов'янофіли» в середовищі російської інтелігенції були борцями за національну самосвідомість… Одна й та ж особистість могла розглядатись як героїко-патріотична — зі сторони держави, і як реакційна — з позицій національно-налаштованих прошарків населення провінцій Російської імперії.

У нашому науковому пошуку ми намагались відшукати історичну достовірність фактів та подій, які б нам допомогли розкрити особливу людську активність та яскравість епохи, а також складний взаємозв'язок духовних особистостей.

Як відомо, до складу Російської імперії згідно з маніфестом Олександра I Грузія добровільно увійшла у 1801 р., убезпечившись загарбання султанської Туреччини і шахської Персії. У ній була встановлена та ж система управління, як і в інших губерніях Росії. В управлінні почали панувати силові військові методи, діловодство і офіційне спілкування запроваджувалось виключно російською мовою. Будь-які серйозні прояви національної самобутності переслідувались. Все це не могло не призвести до закономірного загострення національного питання і сплеску національної самосвідомості корінного населення. Національні процеси супроводжувались ідеологічною конфронтацією з офіційною державною релігією у формі запозичення і розповсюдження ідей духоборства, молоканства, баптизму. Зрозуміло, що найблагодатніший грунт їх підспудного поширення складали саме «молоді єретики», православний дух яких був ще не загартованим, а рівень освіченості вже дозволяв зрозуміти близькість протестантської ідеології національним інтересам. До того ж протестантська ідеологія сприймалась «молодими єретиками» як противага і великодержавному шовінізмові (відстоювала національні ідеї), і жорсткій регламентації внутрішнього семінарського життя (відповідала особистим інтересам молодих семінаристів). Вона поширювалась справді підспудно: для багатьох її адептів була характерною безсумнівно тривка прив'язаність до ортодоксального християнства і, разом з тим, — однозначно критичне до нього ставлення з позиції протестантизму.

Так чим же міг збентежити юних семінаристів М. І. Булгаков, вимога усунення якого повторюється в «Заяві учнів Тифліської духовної семінарії екзарху Грузії, 1 грудня 1893 р.»[17, с. 174–175] аж тричі?! Як випускник КДА, розподілений Св. Синодом до Тифлісу, він просто мав за непорушний обов'язок нести туди православний дух, державність, дисципліну, порядок. За це він, напевно, і був «несимпатичним» для семінаристів. Обов'язок — зобов'язує. Те, що було в Києві допустимо і дозволяло певну легковажність, притаманну М. І. Булгакову, (хоча і тут національне питання постійно тліло), в Тифлісі — вимагало пильності, жорсткості, однозначності як організаційної, так ідейної та духовної.

Здатність випускника діяти однозначно-професійно закладається непересічним навчальним закладом, його духом. Нагадаємо, хто уособлював духовне загартування Миколи Івановича Булгакова, який, навчаючись в академії, славився легковажністю, веселою вдачею, лібералізмом [28, с. 177].

Його вчителями були такі відомі на той час викладачі богословських та філософських дисциплін, як Д. І. Богдашевський, П. І. Ліницький, брати М. О. та Я. О. Олесницькі, М. І. Петров, С. Т. Голубєв, Ф. І. Тітов. З деякими прямо спілкувався свого часу і юний Михайло Булгаков. Не тільки їх публічні лекції в студентських аудиторіях, але й усе їх життя було прикладом самовідданого служіння духу науки, обов'язку і православ'я. Вражають ті страшенні матеріальні незгоди, які випали на долю багатьох з них, хто мав нещастя дожити до революційних подій 1917 р. Про їх душевну кризу можна лише здогадуватись.

Великий вплив на формування світоглядної позиції М. І. Булгакова як апологета православ'я у Тифлісі, справив його рідний брат Афанасій Іванович Булгаков, який закінчив КДА у 1885 р. Сферу наукових інтересів останнього складали саме проблеми історії та аналізу різних течій протестантизму. У 1887 р. за свою працю «Нариси з історії методизму» він отримав ступінь магістра богослов'я, а у 1889 р. перейшов працювати з кафедри стародавньої громадянської історії (до якої, зі спогадів В. П. Рибинського, ніякої прихильності не мав) на кафедру історії західних конфесій. Вся подальша наукова діяльність А. І. Булгакова була пов'язана з вивченням як європейських течій протестантизму, так і їх трансформації на вітчизняному ґрунті.

Розробка проблематики різних течій протестантизму та їх критики з позицій православ'я з необхідністю входила тоді в коло обов'язкових державних наукових замовлень, адресованих викладачам і навіть здібним студентам, оскільки КДА була форпостом панівної державної церкви. Зокрема, прикладом може слугувати показовий факт із життя першого професора філософії університету Св. Володимира, бувшого вихованця КДА Ор. М. Новицького (1806–1884), який ще студентом вперше в Росії написав працю про духоборів, попередників баптизму. Виявляється, Київський Митрополит Євгеній (Болховітінов) в той час розшукував здібного студента останніх курсів КДА з метою дати йому тему магістерської роботи по духоборам і звернувся до ректора Академії Інокентія (Борисова) порекомендувати йому такого. . Так згодом з'явилась перша праця Новицького

«Про духоборів», яка була одночасно першим серйозним вітчизняним дослідженням їх вірувань, побуту та звичаїв.

І цей факт, як і багато йому подібних, надзвичайно вразив нас свого часу. Так от звідки походить авторитетна впливовість навчального закладу, — подумали ми. Наявність у закладі талановитих викладачів і студентів ще не є достатньою підставою для неї. Треба, щоб поза вузівськими владними структурами викладачі і студенти були досить освіченими для розуміння, визнання і підтримки яскравих представників навчального закладу. А сьогодні цього не скажеш навіть про внутрішньо закладові взаємини між викладачами різних дисциплін. Начебто всі публікуються і опубліковане ними звуть наукою. Але декотрі, напевно, забувають, що їх можуть і прочитати. ... І читаємо, і червоніємо інколи за своїх колег. І розуміємо, що подібна «науковість» була і є абсолютною недопустимою у дійсно авторитетному закладі освіти. Думаємо, це розуміють і мудрі представники владних структур, котрі б вважали за честь зробити хоча би раз те, що вдалося їх попередникам ще в XIX столітті.

Але повернемося до нашого сюжету. Якщо оглянути список наукових праць А. І. Булгакова, які протягом 20-ти років його невпинної роботи в КДА періодично друкувались в часописі «Труди КДА», то він складає майже 60 робіт, 40 з яких присвячені історії та аналізу протестантизму. Серед цих робіт, до речі, є такі, як «Баптизм»[5] і «Про молоканство»[6], написані у 1890–1891 рр., тобто у ті роки, коли його молодший брат М. І. Булгаков був студентом останнього курсу КДА і міг вже цілком серйозно цікавитись тими ж проблемами, які хвилювали його старшого брата. Тобто Афанасій Іванович разом з іншими згадуваними професорами Київської духовної академії ґрунтовно посприяв фаховому і духовному загартуванню Миколи Івановича Булгакова. Саме вони уособлювали для нього духовну міць і авторитет його навчального закладу.

Хоча нам і не вдалося точно встановити час вибуття М. І. Булгакова з Тифліської семінарії, про його подальшу долю відомо із вже цитованих спогадів В. П. Рибинського. «Із Тифлісу Булгаков перейшов на посаду місіонера, перехід цей йому здійснив Кутепов (однокурсник Булгакова, секретар Петербурзької консисторії Синоду — *авт.*). Місіонером він виявився поганеньким, і принаймні для столиці непідходящим, тому йому невдовзі довелося виїхати до Новочеркаська, де він і скінчив свої дні» [29, с. 178].

Слід підкреслити, що спомини В. П. Рибинського позначені досить суб'єктивним та інколи недоречно суперкритичним підходом до характеристики своїх викладачів і товаришів, зокрема М. І. Булгакова. Сам В. П. Рибинський не потрапляв в екстремальні умови, у яких побував М. І. Булгаков. Умови ці, як ми бачили, — досить специфічні. З одного боку, їх складає середовище начебто єдиновірців, братів по вірі, котрі мали розуміти і приймати свого старшого брата. З другого, — віра цих єдиновірців виявилась деформованою сторонніми, зокрема протестантськими впливами, які закономірно жорстко вражали М. І. Булгакова і вимагали відповідної ідейної реакції. З третього, — «єретизм» єдиновірців був позначений певними особливостями національного характеру і національних устремлінь, що відбивалось на їх поведінці, котра, закономірно, вимагала відповідних організаційно-адміністративних реакцій. З четвертого, — дух багатьох був вже інфікованим атеїстично — войовничими соціал-демократичними ідеями, котрі збуджували не лише до протистояння ортодоксально-державницькій позиції М. І. Булгакова, але й до відкрито революційного наступу.

Зрозуміло тепер, чому, побувавши у вище змальованих жорстоких умовах і загартувавшись в них, М. І. Булгаков вирішує стати професійним місіонером. Можливо, він і справді не проявив своїх місіонерських здібностей в столиці, оскільки там не було великої необхідності когось переконувати, обертати, чи завертати у православну віру.

Трагічна душевна драма, пережита М. І. Булгаковим у Тифлісі знайшла своє відображення у знайденому нами, до цього невідомому булгаковознавцям, листі М. І. Булгакова з Тифлісу до Києва Миколі Івановичу Петрову — ординарному професору КДА, викладачу естетики, теорії словесності та історії західних літератур, який протягом декількох десятиріч завідував ним же створеним музеєм церковно-археологічних старожитностей при Київській духовній академії. Але, насамперед, Петров був близькою людиною для сім'ї Булгакових — спочатку вчителем Афанасія Івановича, а пізніше — старшим другом і колегою. З народженням першої дитини в родині — сина Михайла, майбутнього письменника — Петров став для нього хрещеним батьком. Саме такій близькій людині адресує свого листа молодший брат, датуючи його 16 квітня 1894 р., тобто згодом після грудневих заворушень 1893 р. В ньому М. І. Булгаков пише: «. . . Я таки частенько згадую Вас, Миколе Івановичу, тут у Тифлісі, особливо після подій 1–4 грудня, які спонукали закриття нашої злополучної семінарії. Мимоволі приходить на згадку Ваша простота, відкрита щирість, — коли постійно бачиш навколо себе лицемірно-східну ласкавість, за якою приховується між тим вогонь ненависті до тебе і найлютіша злість. Адже негідники-семінаристи, які виявились згодом заворушниками, перед бунтом були надто поважливі та люб'язно ввічливі перед семінарським начальством: ніяким чином не можна було передбачити, що в душі цих людей криється прагнення скинути з себе ярмо семінарської дисципліни; ніяким чином не можна було передбачити, що ці пани нап'ються, щоб бути більш наглими під час заворушень і сміливішими, щоб порозумітись з Екзархом Грузії та єпископом Олександром. Так ось, які ці східні люди! А до чого феноменально вони можуть брехати та вперто замовчувати свою провину? Людина, яка незнайома з цим народом, може навіть не повірити деяким фактам, котрі характеризують грузино-імеретин з цієї сторони. М. Булгаков» [20].

Душевний розпач, палка образа, і трагічне розчарування в людях, яких М. І. Булгаков усіма засобами прагнув наставити на істинний шлях, звучить у щойно процитованому листі. Але в ньому відбились лише емоції, котрі, мабуть, призвели до загострених оцінок, певних перебільшень та необґрунтованих узагальнень. Найбільшого обурення автора, як бачимо, викликає не стільки прагнення учнів скинути ярмо семінарської дисципліни, скільки їх нещирість. Так, М. І. Булгаков добре розуміє, як важко молодій людині жити з цим ярмом; він і сам це пережив, навчаючись до КДА в Курській семінарії. Але ж нещирість. . . . Вона не знає виправдань з точки зору вихованця КДА, який шукає духовної підтримки у М. І. Петрова — людини відкрито щирої. Брехня, обман, лукавство — означає для М. І. Булгакова відсутність честі, що несумістимо зі справжньою духовністю — чи то релігійною, чи то громадянською.

Тема деструктивної спрямованості лукавства, нещирості і нечесності лунає і в його промові «Баптизм як секта, небезпечна для держави» [10]. Характеризуючи ватажків баптизму, зокрема, Томаса Мюнцера, Іоанна Лейденського, М. І. Булгаков звертає увагу слухачів на абсурдні висновки, зроблені ними з тлумачень Біблії. «...Як тлумачилась ця біблія?... Найбільш обурливим чином: у цій святій книзі баптисти прагнули знайти грунт для своїх найдикіших і аморальних вчинків»[10, с. 18]. Детальний опис останніх у промові містить справді вражаючі факти з життя Іоанна Лейденського — його біснуваті претензії на побудову баптистської держави, на світове царювання та управління народами, безмежний фанатизм, багатожонство, звірство і розпутність тощо [див.: 10, с. 18–22]. Томас Мюнцер також зображається як «відчайний релігійний фанатик, схильний до брехні, обману і лукавства для досягнення мети» [10, с. 11]. Мета ж, як вважає промовець, — у них одна: «зламати основи державні, сімейні і моральні в самому широкому смислі цього слова»[10, с. 11]. А вже потім — побудувати «нове суспільство», яке б ґрунтувалось на подоланні опозиції «бідність — багатство», на спільності майна між послідовниками ідеології баптизму. Тобто і в деструктивній і в конструктивній частинах програма баптизму цілком підводиться М. І. Булгаковим під революційно-соціалістичні спрямування [див.: 10, с. 15]. Справді, зазначає промовець, — баптизм «... з релігійної секти зробився громадянською»[10, с. 11], а самі баптисти отримали «клеймо підбурювачів і революціонерів» [10, с. 22]. Тому головний наголос у Промові робиться на «...політичних устремліннях баптизму, — устремліннях, на які до останнього часу так мало зверталось уваги і світською і духовною владою»[10, с. 10]. Як це не дивно, але витоки сталінізму грунтуются і в баптизмі.

Знаючи, що майже через півстоліття після описуваних вище Тифліських подій племінник М. І. Булгакова — Михайло Афанасійович Булгаков читав промову про баптизм, ми подумали, а чи не могли її мотиви відобразитись у п'єсі «Батум», зокрема у вже згадуваній промові Ректора семінарії у Пролозі? Дійсно, деякі важливі паралелі між цими промовами є справді помітними. Ректор також робить наголос саме на політичних, антидержавних устремліннях різноплемінних мешканців батьківщини, злочинців, «які сіють зле семя у нашій країні»[7, с. 670]. Підспудно увійшов у ректорську промову і помічений ще Миколою Івановичем той історичний факт, що розпутний Іоанн Лейденський свого часу став героєм опери композитора Мейєрбера «Пророк» [10, с. 20]. «Народні спокусники і лжепророки, — продовжує Ректор, — прагнучи підірвати міць держави, поширюють скрізь отруйні хибно-наукові соціал-демократичні теорії, які, подібно гострим струменям злого духу, проникають в усі щілини нашого народного життя»[7, с. 670]. Тому і смисл придуманої М. А. Булгаковим партійної клички Й. В. Джугашвілі «Пастир» [7, с. 679], про яку йдеться у другій картині п'єси, не повинен сприйматися буквально. Пастирство Сталіна має спільну природу з пророцтвом І. Лейденського і сенс Сталінського псевдоніму закладається Майстром з врахуванням його відношень до вищеназваних обставин.

Як бачимо, промова Ректора — високого церковного ієрарха — зовсім не зводиться до суто релігійних звинувачень молодого відступника.

Вона, перш за все, засуджує антидержавницькі устремління. Так по відношенню до якої держави звинувачується Й. В. Сталін у своїх деструктивних політичних спрямуваннях? Текст промови Ректора начебто містить пряму відповідь; саме до Російської імперії. . . . Але врахування контексту (хоча би статті М. І. Булгакова про баптизм) дає підстави стверджувати і факт наявності в ректорській промові з «Батуму» певного завуальованого підтексту. Бездуховність, нещирість, підступність заполітизованого, революційно-заколотницького баптизму (зрозуміло, і не лише його), — «віддають, — читав М. А. Булгаков у праці свого дядька, — будь-яку державу (підкреслено нами — *авт.*) на поталу ватажків баптизму і роблять її іграшкою в їх руках; цією іграшкою будь-хто з наставників баптизму буде грати за своєю примхою. . . »[10, с. 24]. Так і сталося в дійсності, абсурдність, яку добре знав, яскраво змальовував у своїх творах і болісно переживав Майстер.

На схилі свого короткого життя, коли палка спрага гучної світової слави вже вщухла разом з іншими поривами молодості, приглушена заборонами публікацій і постановок, розчаруваннями і хворобами, з'явилась тиха ностальгія і безнадійна печаль. Дух письменника чинив опір їм, але його думка все частіше поринала у минуле. У споминах до нього приходять образи близьких людей, епізоди київської юності, краєвиди рідного Києва. За декілька тижнів до смерті, прикутий до ліжка і повністю вже сліпий, письменник просить свого друга Гдешинського відповісти «найдетальнішим чином на його питання про київське життя часів їх молодості — звичайні програми концертів у Купецькому саду, склад бібліотеки КДА, яку вони відвідували тощо» [33, с. 645].

Отже, науковий пошук авторів даної публікації дозволив пролити світло на ряд аспектів маловивченої біографії М. І. Булгакова, поставив нові питання і заставив ще раз задуматись над духовним взаємовпливом у просторі і часі творчих особистостей.

У ході роботи над цими питаннями, авторами була зібрана унікальна колекція світлин, пов'язана з історією Київської духовної академії та сім'єю Булгакових. Нижче ми приводимо деякі світлини з цієї колекції.

Профессорская корпорація со студентами XXIV курса Академіи (1869 г).

Микола Іванович Булгаков

Двір Київської Духовної Академії, XIX ст.

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«Мир моих идей впереди»

к 150-летию со дня рождения Владимира Ивановича Вернадского

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В марте 1863 года, в Санкт-Петербурге в семье профессора экономики и истории родился будущий великий ученый Владимир Иванович Вернадский.

Конец XIX и начало XX века характеризуются революцией в естественных науках: открыты невидимые рентгеновские лучи, открыта радиоактивность, Эйнштейн размышляет о природе света и произносит слово «относительность», Циолковский вычисляет космические скорости...

Происходит духовная революция в философии и в искусстве. Владимир Соловьев утверждает, что в творческом акте возможно преобразовать реальность, сделать ее материю менее плотной, поднять частоту вибраций энергетических воздействий.

Кандинский и Малевич начинают эксперимент в живописи по поиску способов передачи при помощи материальных средств — холста, картона, красок — абстрактных понятий, таких как «вечность», «бесконечность», «движение», ... ощущений и чувств, приходящих к нам из неосязаемых миров. Чюрленис и Врубель пишут пророческие полотна. Композитор Скрябин ищет общие корни звука, цвета и поэтического ритма. Культура «серебряного века» пропитана ощущением инобытия, влияния космоса на земную жизнь.

В 1885 году Вернадский заканчивает физико-математический факультет петербургского университета, успешно защищает диссертацию на степень кандидата естественных наук и получает должность хранителя Минералогического кабинета университета. Ученый организует экспедиции на Урал, Забайкалье, Фергану и Кавказ, где исследует почвы и породы, собирает коллекции минералов. Особое внимание уделяет ферганским рудам, содержащим радий. Как предтеча нового естественнонаучного мировоззрения появляется его запись о минералах: «Кто знает, может быть, есть законы в распределении минералов, как есть причины возможности образования той или иной реакции именно в этом месте, а не в другом...». Затем, Вернадский формулирует задачу всей своей жизни, он пишет: «Минералы — остатки тех химических реакций, которые происходили в разных точках земного шара, эти реакции идут согласно законам, нам неизвестным, но которые мы можем думать, находятся в тесной связи с общими изменениями, какие претерпевает Земля как звезда. Задача — связать эти разные фазисы изменения Земли с общими законами небесной механики...». Вернадский приходит к новому пониманию биосферы (поверхности геологической оболочки Земли на которой сосредоточена область жизни) и вводит понятие живого вещества (совокупности всех живых существ биосферы). Он пишет: «Космические излучения вечно и непрерывно льют на лик Земли мощный поток сил, придающий совершенно особый, новый характер частям планеты, граничащей с космическим пространствам... Вещество биосферы благодаря этим излучениям проникнуто энергией, оно становится активным, собирает и распределяет в биосфере полученную в форме излучения энергию, превращая ее в энергию в земной среде свободную, способную производить работу. Земная поверхностная оболочка не может, таким образом, рассматриваться как область только вещества; это область энергии, источник изменения планеты внешними космическими силами». Живое вещество, таким образом, находящееся в постоянной изменчивости в зависимости от небесной механики, космических воздействий на Землю, оказалось в целом подчиненным мере и весу. Первым в мире Вернадский найдет основные формулы для живого вещества, вычислит скорости растекания его по поверхности планеты, энергию «давления жизни».

В начале XX века на культурную революцию, охватившую все стороны человеческой жизни, наложилась революция социальная. Социальная революция наряду с идеями справедливого переустройства мира несла и огромные разрушения. Николай Бердяев писал: «...В апокалиптическом времени величайшие возможности соединяются с величайшими опасностями. То, что происходит с миром во всех сферах, есть апокалипсис целой огромной космической эпохи, конец старого мира и преддверие нового мира. ...В поднявшемся мировом вихре, в ускоренном темпе движения все смещается с своих мест, расковывается стародавняя материальная скованность. Но в этом вихре могут погибнуть и величайшие ценности, может не устоять человек, может быть разодран в клочья».

С конца 1917 по 1921 год Вернадский живет в Украине — Киеве, Полтаве, Харькове, Крыму. В 1918 году его избирают первым президентом Украинской Академии Наук. Вернадский разрабатывает устав Академии, организует научную библиотеку Академии и много работает. Он ведет дневник. В дневнике этих лет часто встречается фраза «работаю над живым веществом» и наряду с этим другие записи, о революции: «Масса замученных и избитых, истерзанных людей... Какой ужас и какое преступление. И какая без героев и каторжная русская революция». Или вот, после входа в Полтаву большевиков: «Яркий идеал сытых свиней: обжорство, пьянство, зрелища, свадьбы. Безделье царит. Семечки, кинематограф, хироманты, внешний лоск, грабят где можно, трусость перед вооруженными и смелость перед безоружными. Тяжела социалистическая революция своим насилием...» И тут же о том, что дает силы жить: «Работаю много над живым веществом. И здесь нахожу опору... Надо найти и нахожу опору в себе, в стремлении к вечному, которое выше всякого народа и всякого государства. И я нахожу эту опору в свободной мысли, в научной работе, в научном творчестве... Пишу. Дома все спят. А на улице треск пулемета. Человек привыкает ко всему».

В 1921 году Вернадский возвращается в Петроград. Его арестовывают, помещают в вонючую камеру, следователь грубо ведет допрос. Протокол не ведется — следователь неграмотный... Друзья Вернадского пишут Луначарскому и Ленину и Вернадского освобождают.

В мае 1921 года, в Петрограде в Доме литераторов Вернадский читает лекцию «Начало и вечность жизни». В городе не работают заводы и фабрики, не работает транспорт, надвигается голод и многих удивляет чтение этой лекции. Зачем и кому она нужна? Сейчас, через почти сто лет, мы можем сказать, что эта лекция была нужна, она доказывала, что мир мысли существует над конкретным временем и государством. Вернадский уже в то время задумывается о ноосфере (сфере мысли).

В конце 1921 года был получен первый препарат чистого радия из отечественного сырья — ферганской руды. А уже в начале 1922 года был создан Государственный Радиевый Институт. Вернадский становится заведующим геохимическим отделом этого института. Еще в 1911г. Вернадский специально приезжал в Париж с целью привлечь М. Кюри и А. Лакруа к составлению карты радиоактивных минералов земной коры, организации международного проекта по исследованию радиоактивности. Мария Кюри поддержала идею Вернадского, но проекту этому не суждено было осуществиться.

В 1922 году Вернадский получает приглашение прочесть цикл лекций в Парижском университете и уезжает в длительную научную командировку. В Париже ученый читает лекции, работает в Музее естественной истории и в Институте Кюри. Из Парижа он пишет своему другу и коллеге Б. Личкову: «Все здесь переполнено теорией Эйнштейна, новыми достижениями в атомных науках и астрономии. Я весь погружен в эти новые области... Мне кажется, сейчас переживается такой момент, равного которому не было в истории мысли».

Научная мысль определила все явления относительными.

Относительное земное время, привязанное к вращению Земли вокруг Солнца, не очень подходило для описания процессов в биосфере и Вернадский вводит понятие биологического времени. Вернадский считал, что живые существа не живут во времени, а длят его.

Вернадский определял основным свойством живого вещества его размножение, считал размножение основным видом его движения. Биологическое время, таким образом могло рассматриваться как следствие биологических явлений и исчисляться длительностью жизни популяций. Главными свойствами биологического времени-пространства Вернадский называл необратимость и диссиметрию (момент возникновения жизни на Земле Вернадский связывал с возникновением асимметрии в строении белковых молекул). Над вопросом геометрии биологического времени — пространства (именно время-пространства, а не физического пространства-время) Вернадский размышлял до конца своей жизни. Ученый считал, что геометрия пространства живого вещества не может быть геометрией Евклида, возможно Римана... Он понимал, что математика еще не готова описать геометрию этих пространств и воспринимал их интуитивно. Используя терминологию сегодняшнего дня мы можем сказать, что Вернадский исследовал три аспекта Природы Вселенной:

— материю
— информацию
— структуру.

Он прозревал синтез биоэнергоинформации Космоса. Разве может существовать «бессознательная» материя, неспособная воспринимать и отвечать на информацию? Разве кристаллы, зарождаясь в результате определенной химической реакции в определенном месте под влиянием определенных космических лучей, не ростут и не разрушаются со временем? Известно, так же, что изъятый из своей среды кристалл и помещенный в человеческое биополе может помутнеть, поменять цвет, разрушиться. Кристаллическая решетка может разрушиться и от воздействия диссонансной музыки. Вернадский думал над этим еще в 1905 году. Позже, в Крыму он писал в дневнике: « Перед 1905 годом, когда я все глубже уходил в полиморфизм и кристаллографию. Хотел выявить кристаллизацию перенасыщенных растворов звуками. Заказал камертоны — остались в Москве и заржавели. Идея — созвучие, резонанс. Никто не исследовал. Может быть меняются и комбинации». Значит кристаллы имеют свой уровень сознания, способность получать и реагировать на информацию. Информационные поля, — левые и правые безмассовые вихри, геометрия левой и правой кривизны. . . В том же 1905 году Вернадский начал но не довел до конца исследования: «. . . в растворах лево — и правовращающихся веществ — мыльные пленки: спиральные фигуры равновесия Плато — левые и правые. . . ». Проблема левого и правого будет интересовать Вернадского все время. К математику Лузину он обратиться с вопросом можно ли математически описать левое и правое. Вернадский так же исследовал структуры сложных систем живой природы, их всевозможные и бесчисленные связи. Из записей в его дневнике: «Утром ходил на луг и набрал цветов. . . На Verbascum пчела, полная пыльцы, захваченная каким-то оригинальным пауком. . . Проявление своеобразного строения живого вещества и хода перемещения химических элементов», или «. . . о хамсе — количество ее как будто совпадает с геологическими тепловыми периодами. Ее космическая роль — переработка планктона: поддерживает других хищных рыб — макрелей и т. д. 10-летний период, как солнечные пятна?» Дневники Вернадского — это непрерывный эксперимент, наблюдения, вычисления и размышления, наблюдение жизни во всех ее аспектах. Наблюдая и исследуя синтез материи, информации и структуры, Вернадский понимал, что и каждый аспект в отдельности нужно изучать экспериментально, вводя число в его характеристики. Он писал: «Сила моей работы не только в том, что я работаю своей мыслью в почти незатронутой области. . . не теряя связи с фактами, я ввожу число в область, ранее его лишенную».

Изучая косную материю и живое вещество биосферы, Вернадский задумался о природе и значении мысли. Он изучал историю научной мысли и пришел к выводу, что научная мысль есть продукт творческой деятельности ученого, научную мысль творят личности. Вернадский написал: «Научная мысль сама по себе не существует, она создается человеческой живой личностью, есть ее проявление. В мире реально существуют только личности, создающие и высказывающие научную мысль, проявляющие научное творчество — духовную энергию. . . ». Вернадский пришел к заключению, что историю научной мысли «нельзя рассматривать только как историю одной из гуманитарных наук. Это история есть одновременно история создания в биосфере новой геологической силы — научной мысли, раньше в биосфере отсутствовавшей». Так родилась работа «Научная мысль как планетное явление» и определение эволюции как движения от биосферы к ноосфере. Вернадский считал, что разум — это та сила, которая делает эволюцию целесообразной. Исследуя процессы зарождения кристаллов под теми или иными космическими лучами в следствии определенных химических реакций, Вернадский предполагал наличие химизма космических лучей. Лекции Менделеева произвели на него большое впечатление, но известная таблица химических элементов могла быть только началом науки об элементах. Возможно, массовые числа элементов могут исчисляться сотнями, в зависимости от интенсивности космических излучений? Космические лучи попадая на Землю от различных космических тел, несут информацию об этих телах, по сути являясь «сознанием» космических объектов. Чем выше частота по торсионной шкале и меньше квантовые интервалы космических излучений, тем с большим ускорением происходят информационно-мыслительные процессы на Земле. Можно предположить, что Вернадский рассматривал мысль как энергию, мысленную энергию, которую нужно начать изучать. Гармоничное устройство космоса предполагает повышение уровня вибрационных процессов в биосфере Земли и ускорение процессов физического и ментального развития при повышении активности Солнца. В случае дисбаланса получаемых планетой космических излучений и их восприятием (осознанием) неизбежны природные катаклизмы той или иной степени. Понимая это, Вернадский считал три задачи наиболее насущными — развитие научных исследований, просвещение и «гигиену мысли» («надо не позволять себе думать о всем дурном»). Так рождалось учение о жизни-сознании-мысли космического масштаба, осмысление которого еще впереди.

В 1926 году Вернадский принимает решение вернуться в Россию.

С 1927 года и до самой смерти Вернадский директор Биогеохимической лаборатории при Академии наук СССР в Москве. Он разрабатывает программу ядерных исследований в СССР, составляет карту минерально-сырьевой базы страны, как академик принимает активное участие в жизни Академии Наук СССР.

Только почему-то его работы по конкретным исследованиям и опытам публикуют, а работы связанные с его новой научной парадигмой, новым мировоззрением в естествознании, под разными предлогами кладутся под сукно. Так, например, работа «Научная мысль как планетарное явление» была впервые опубликована только в 1977 году.

В 1936 году страну окутала тьма, ученых перестали выпускать заграницу, начались массовые аресты.

Вернадский продолжает вести дневник. Когда мы читаем этот дневник, мы видим человека, который хорошо понимал, что происходит вокруг, который болезненно воспринимал происходящие вокруг разрушения. В дневнике есть записи с характеристикой Сталина, есть запись о том, что в стране создана «лагерная производственная сила», многие страницы дневника по тем временам означали расстрельную статью. В своем дневнике Вернадский пытается анализировать, исследовать происходящее и, возможно, он думает о том, что его будут читать потомки... Дети Вернадского, его сын Георгий — профессор истории и дочь Нина-врач, жили в

Америке и Вернадский с женой не имели возможности с ними видеться. В 1937 году Вернадский переживает кровоизлияние, в результате которого происходит временный паралич правой руки, врачи определяют неясные процессы, происходящие в сердце.

Но, несмотря на все тягости и горести конкретного времени и своей личной жизни, Вернадский исповедовал исторический оптимизм. Вернадский верил в необратимость научного знания: «Процессы, подготовлявшиеся много миллиардов лет, не могут остановиться. Отсюда следует, что биосфера неизбежно перейдет в ноосферу, т.е. в жизни народов произойдут события , нужные для этого, а не этому процессу противоречащие». Живя в своем физическом теле в конкретное время конкретного пространства, в своих мыслях Вернадский жил в будущем и своей работой приближал это будущее.

С большим волнением встретил Вернадский известие о начале войны. 15 июля 1941 года Вернадский впервые в жизни выступил по радио, в своей речи он сказал: «В моих исследованиях по радиоактивности я сотрудничал с великим Резерфордом, с Джоли — основателем радиогеологии, со Спенсером, Мэллори и другими учеными, в настоящее время я сотрудничаю с Панетом, который нашел приют в Англии от преследования гитлеровских фашистов... В эти дни тяжелой борьбы против фашистских захватчиков, я приветствую вас, мои коллеги по науке, и я глубоко убежден, что наш общий враг будет скоро разбит и справедливость восторжествует». Вернадский обращался к ученым Англии и всего мира, считавшим своим долгом своими мыслями и работой противостоять фашизму. Вернадский предсказал победу советской армии в Сталинградской битве и связал ее с началом ноосферы.

В 1943 году, к восьмидесятилетию, Вернадский был награжден Сталинской премией. В своей телеграмме Сталину по этому поводу он писал: «Прошу из полученной мною премии Вашего имени направить 100 000 рублей на нужды обороны, куда Вы найдете нужным. Наше дело правое и сейчас стихийно совпадает с наступлением ноосферы — нового состояния области жизни, основы исторического процесса, когда ум человека становится геологической планетной силой. Академик В. Вернадский».

В 1943 году ушла из жизни жена, друг и сотрудница Вернадского — Наталия Егоровна Вернадская (Старицкая), с которой ученый прожил 56 лет жизни. Ученый подал прошение разрешить ему уехать к детям в Америку. На прошение пришел отказ. Вскоре у Вернадского произошло кровоизлияние и через две недели, 6 января 1945 г. он покинул этот мир. Из дневника Вернадского: «Страха смерти у меня нет и никогда не было. Чувство мгновенности жизни — чувство вечности и чувство ничтожности понимания окружающего! И себя самого!», «В сущности та бесконечность и беспредельность, которую мы чувствуем вокруг в природе, находится и в нас самих... «Час» жизни — как мало времени и как бесконечно много содержания».

О Вернадском нельзя рассказать на шести страницах, в этом коротком эссе я не упомянула о многих важных проблемах, которые поднял ученый, таких, например, как проблема автотрофности человечества. К счастью, у нас сохранились научные работы Вернадского и его дневники, мы можем читать их и сопоставлять сегодняшние научные достижения с его мыслями и теориями. Цитаты, приведенные мною в данном эссе взяты из книг: В. И. Вернадский «Дневники 1917–1921», Наукова Думка, Киев, 1997 г., В. И. Вернадский. Из-во «Планета», Москва, 1988 г., Вернадский. Издательский Дом Шалвы Амонишвили, Москва, 2001 г.

Цитаты из научных работ приведены по изданию: В. И. Вернадский «Биосфера и Ноосфера», Айрис-Пресс, Москва, 2009 г. Мной так же была использована книга Г. П. Аксенова «В. И. Вернадский. О природе времени и пространства», Из-во «Красандр», Москва, 2010 г.

Заканчивая это эссе о мыслителе, ученом и гражданине, мне хочется привести еще одно высказывание Вернадского. В один из самых тяжелых периодов своей жизни, выступая с докладом в Академии Наук, Вернадский сказал: «Мы переживаем не кризис, волнующий слабые души, а величайший перелом научной мысли человечества... Может нечто подобное было в эпоху зарождения эллинской научной мысли, за 600 лет до нашей эры. Стоя на этом переломе, охватывая взором раскрывающееся будущее, — мы должны быть счастливы, что суждено в создании такого будущего участвовать».

Когда на последней картине земной
выцветет кисти след,
засохнут все тюбики и помрет
последний искусствовед,
мы отдохнем десяток веков,
и вот в назначенный час
Предвечный Мастер Всех Мастеров
за работу усадит нас.
Тогда будет каждый, кто мастером был,
на стуле сидеть золотом
и по холстине в десяток миль
писать кометным хвостом.
. . . .
И только Мастер похвалит нас,
и упрекнет только Он,
и никого тогда не прельстит
ни денег, ни славы звон:
лишь радость работы на новой звезде —
дана будет каждому там...

Р. Киплинг. «Послание»

24.03.2013

Interdisciplinary Studies of Complex Systems
No. 2 (2013) 152

Зміст

Interdisciplinary Studies of Complex Systems
No. 2 (2013) 153

Contents

Тематика та мета журналу

«Міждисципрлінарні дослідження складних систем» — це рецензований журнал із вільним доступом, що публікує дослідницькі статті, огляди, повідомлення, дискусійні листи, історичні та філософські студії в усіх областях теорії складних систем для впровадження взаємодії між науковцями з різних галузей математики, фізики, біології, хімії, інформатики, соціології, економіки та ін. Ми бажаємо запропонувати істотне джерело актуальної інформації про світ складних систем. Журнал має стати частиною наукового форуму, відкритого та цікавого як для експертів з різних областей, так і для широкої аудиторії читачів: від студентів до досвідчених дослідників. Журнал надає можливість для науковців з різних галузей презентувати нові ідеї, гіпотези, піонерські дослідження. Особливо запрошуються до публікації автори наукових статей та (але не тільки) наукових оглядів, проте статті з історії та філософії науки, інформації про наукові події, дискусійні повідомлення також вітаються.

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