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Mathematical Modelling of Radionuclide Migration and Reliability of Natural Systems

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As a result of the accident at the Chernobyl Nuclear Power Plant (ChNPP) in 1986, about 3.5 million hectares of forests in Ukraine fell under radioactive contamination, and the entire forests in Ukraine occupy 9.9 million hectares. Territories of other countries were significantly affected. In addition to Ukraine, the Republic of Belarus and the Russian Federation, the influence of the Chornobyl nuclear disaster was felt by Sweden, Norway, Poland, Austria, Switzerland, Germany, Finland, Great Britain and other countries [1–3]. Another significant radioactive contamination of the territories was the result of the accident at the nuclear power plant Fukushima-1 in 2011. In general, there were more than 100 serious accidents involving the use of nuclear energy in the whole world, and more than 50 occurred after the Chernobyl disaster. Consequently, the problem of radioactive contamination remains relevant, and the results obtained from the analysis of the Chernobyl catastrophe may be useful for modelling and predicting other similar phenomena.

Mathematical modelling is one of the most effective methods for studying the migration processes of radioactive substances in the environment. The Kyiv school of scientists has proposed a methodology for assessing the radiation state of ecological systems based on the use of mathematical box models, the theory of reliability, and radio capacity parameters assessment.

Application of Box Models in Radioecology

Stationary box models are based on the postulate of existing a stable statistical equilibrium in the system "an ecosystem — an organism — an environment". In this case, the distribution of radionuclide activity in each of the boxes is considered uniform.

Figure 1 a) presents a relatively simple ecosystem — a stationary box model consisting of four boxes with given initial radioactivity $\tilde{N}_1$ in the soil and the transition coefficients $(\hat{E}_T)$ between boxes $\hat{E}_1, \hat{E}_2, \hat{E}_3$.

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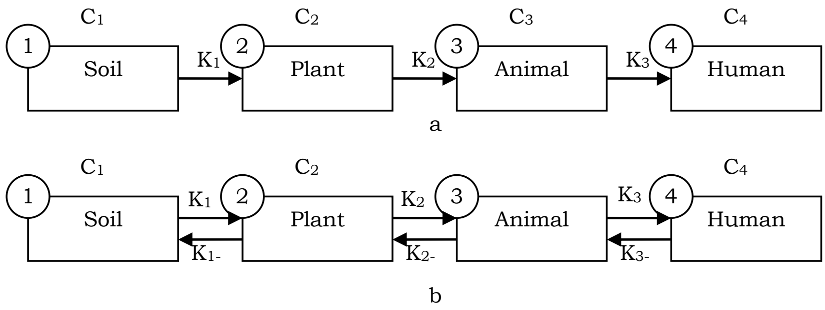

Fig. 1. The simplest stationary (a) and dynamic (b) box models of an ecosystem

According to this data, using the stationary box model, it is easy to calculate the activity of radionuclides in other boxes:

$$\begin{aligned}
\tilde{N}_2 &= \hat{E}_1 \cdot \tilde{N}_1, \\
\tilde{N}_3 &= \hat{E}_2 \cdot \tilde{N}_2 = \hat{E}_1 \cdot \hat{E}_2 \cdot \tilde{N}_1, \\
\tilde{N}_4 &= \hat{E}_3 \cdot \tilde{N}_3 = \hat{E}_1 \cdot \hat{E}_2 \cdot \hat{E}_3 \cdot \tilde{N}_1.
\end{aligned} \tag{1}$$

In addition to simple stationary box models, dynamic box models are widely used; they are based on two basic assumptions:

1) The ecosystem can be divided into several interacting boxes, which eventually undergo radionuclide exchange. Radionuclides that arrive in the box instantly mix the same way in all parts of the box at any given time.
2) The loss of the radionuclides by the box is proportional to the activity of the radionuclides in this box. The transfer of radionuclides from one box to another occurs according to the first order kinetics laws; it is described by a system of ordinary differential equations. In this case, the proportionality coefficient between the specific activity of radionuclides in the box and arrival of radionuclides from this box to any other (the coefficient of radionuclide transfer between boxes) is constant.

Figure 1 b) presents the corresponding ecosystem (dynamic box model), where $\tilde{N}_1, \tilde{N}_2, \tilde{N}_3, \tilde{N}_4$ — dynamic specific radionuclide activities in the model's boxes, $\hat{E}_1, \hat{E}_2, \hat{E}_3$ — direct coefficients of radionuclide transfer between boxes, $\hat{E}_1^-, \hat{E}_2^-, \hat{E}_3^-$ — inverse coefficients. The dynamics of radionuclide activity in such a system is represented by the following system of differential equations:

$$\begin{aligned}
\frac{dC_1}{dt} &= K_2^- C_2 - K_1 C_1, \\
\frac{dC_2}{dt} &= K_1^- C_1 - K_2 C_2 - K_2^- C_2 + K_3^- C_3, \\
\frac{dC_3}{dt} &= K_2 C_2 + K_4^- C_4 - K_3^- C_3 - K_3 C_3, \\
\frac{dC_4}{dt} &= K_3 C_3 - K_4^- C_4.
\end{aligned} \tag{2}$$

If there is a constant inflow of radionuclides into the first box "soil", then another equation is added to the system:

$$\frac{dC_0}{dt} = K_0 C_0, \tag{3}$$

where C_0 is intensity of the radioactivity source at the moment of emission (Bq); K_0 — the coefficient of radionuclide transfer from the source to the first box. In this case, the first equation of the system is supplemented by one more term: $+\hat{E}_0\tilde{N}_0$. Practically for any complex and branched ecosystem, an appropriate system of equations can be compiled and solved (for example, we use the software MAPLE).

The method of box models is the simplest and adequate mathematical method for describing radioecological processes in ecosystems of various complexity [4].

The Concept of Reliability in Radiobiology of Multi-level Biological Systems

Biological objects have extremely high reliability, which greatly exceeds the reliability of any technical systems. This follows primarily from the time of the existence of biological systems, which significantly exceeds the time of failure-free existence of technical systems. As a definition of the concept of biosystems reliability, one can offer the following: the reliability is a fundamental property of biological objects which determines their effective existence and functioning in randomly varying environmental and time conditions. The degree of reliability is the probability of fail-safe existence of systems which can vary from 0 to 1.

Two main types of systems are distinguished in the mathematical theory of system reliability [5]. Let the reliability of a separate element of the system be determined by P_i — the probability of fault-free existence of the element.

The first and the simplest type of the system consisting of many elements is a system of sequential type. Mathematically, the reliability of such a sequential system with n-elements is determined by the formula of probabilities multiplication:

$$P_{\text{series}} = \prod_{i=1}^{n} P_i. \tag{4}$$

It is clear that such a consistent system has extremely low reliability since the failure of at least one element leads to the failure of the entire system. Even the high reliability of P_i elements cannot provide the high reliability of such a sequential multi-element system.

Another type of system is a parallel type system. Such systems can fail only when all their operating elements are in a state of failure. Almost all electrical networks in residential buildings, as well as in industry work according to this scheme.

If the failure probability of one of the elements is P_i, then the probability of failure-free existence will be equal to $1 - P_i$. In a parallel system, the elements work independently, therefore, according to the formula for multiplying probabilities, the failure probability of all n-elements looks as follows:

$$P_{\text{system failure}} = \prod_{i=1}^{n} (1 - P_i)\,. \tag{5}$$

Then the probability of a failure-free existence of such a parallel system is determined by the formula:

$$P_{\text{parallel}} = 1 - \prod_{i=1}^{n} (1 - P_i)\,. \tag{6}$$

Obviously, a system built according to the parallel principles will be highly reliable even if the reliability of its individual elements is insignificant. Such a property of the parallel system lies at the heart of the reservation method, often used while creating highly reliable technical systems, and in the structure of existing biological systems [5–8].

Algorithm for calculating the reliability of ecological systems

The developed models and the theory of ecosystems radioactivity made it possible to introduce a radio capacity factor to determine the state of the ecosystem biota.

The factor of ecological and radiation capacity F_j for a given element of the landscape or ecosystem is defined as follows:

$$F_j = \sum a_{ij} \Big/ \Big(\sum a_{ij} + \sum a_{ji}\Big), \tag{7}$$

where $\sum a_{ij}$ — the sum of the rates of the pollutants transition from various components of the ecosystem to a specific element of the landscape or ecosystem according to box models, $\sum a_{ji}$ — the sum of the pollutants outflow rates from the investigated box J — to other conjugated parts of the ecosystem.

The algorithm for calculating the reliability of ecosystems consists of the following stages:

1) Constructing the box model of the ecosystem that is being studied.
2) Determining the radionuclide transition speed parameters (tracers — ^{137}Cs) between the boxes of the studied ecosystem.
3) In accordance with the formula (7), calculating the radio capacity parameters and reliability of each element of the studied ecosystem.
4) Determining the reliability of the studied ecosystem structure, which can be sequential, parallel or combined.
5) Defining the formula for calculating the overall reliability of the entire ecosystem on the basis of a reliable structure of this ecosystem.
6) Calculating the overall reliability of the entire ecosystem by formula (7) using the elements reliability parameters.

Thus, the method of structuring ecological systems includes four successive procedures:

1) the construction of a box radio-ecological model of the corresponding specific ecological system;
2) determination of its configuration (sequential, parallel or combined);
3) determination of parameters and rates of radionuclide transition;
4) calculation of the reliability of structural elements and the ecological system as a whole.

This method has been applied to modelling the transport of radionuclides and predicting their maintenance in systems for ecosystems "Forest", "place-Lake", "Marsh" and agroecosystems of varying complexity. [9–12].

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