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Social justice as a subjective analysis category. Numerical estimations

Volodymyr Kasianov[1], *Andrij Goncharenko*[2]

Introduction

The notion of *"Social Justice"* [1] is an object of the *"Theory of Collective Solutions Making"* [2], related to the *"Welfare Theory"* [3].

The term of: *"Social Justice"*, being an old notion, lately became extremely popular in some politicians speeches. Especially it is noticeable in the periods of election campaigns, political parties' discussions, politicians' statements.

Nevertheless, it can be definitely said that these people hardy imagine the sense of that category. Regular voters understand the meaning of *"Social Justice"* even weaker and mostly at the emotions level. Therefore, a successful literature formulation substitutes the core of the problem and leads the discussion on some other point.

The task for the authors, in the presented work, is to clarify, to some degree, the issue about *"Social Justice"* on the basis of the old theory of *"Collective Utility"* and proposed and developed by one of the authors *"Subjective Entropy Maximum Principle"* [4-6]. The latter is the keystone of the theory of *"Subjective Analysis"* [4-6] developed after the *"Jaynes' Entropy Principle"* [7, 8]. Hereinafter, we propose the method to analyze the problem numerically and investigate it parametrically.

The synthesis of the *"Utility Theory"* and *"Subjective Analysis"* opens new vision of the problem of *"Social Justice"*.

1) First of all there is an external view on the system; it is objective. If we say about a subjective component, we have to take into consideration the interaction of the socium participants.

2) Next up of the preliminary notes is that the term of *"Social Justice"* evidences about not only individual attitudes, but also their collective feelings, although all their evaluations arise inside the actors psych.

Individual estimations of the *"Social Justice"* criteria aggregate by specified rules into the collective estimations; also, in parallel to the individual and inside the individuals' psyches.

3) The *"Social Justice"* category is a dynamical one. This means that every time criteria variate. That is at the solution of the dynamical problems

[1] Professor, Doctor of Engineering Sciences, Professor of Mechanics Department of the National Aviation University, vakasianov270435@gmail.com

[2] Associate Professor, Doctor of Engineering Sciences, Professor of Aircraft Airworthiness Retaining Department of the National Aviation University, andygoncharenco@yahoo.com

of the *"Social Justice"* a recursion methods are of significance. The time span of the recursion has to be selected with taking into account an entropy time, introduced and considered in the sequence of works related to entropy approaches [9, 10].

Thus, the synthesis of *"Collective Utility"* and *"Subjective Entropy Maximum Principle"* from *"Subjective Analysis"* leads to the variational problems with the additive functionals. The main member of such functionals is the *"Subjective Entropy"* depending upon the considered problem. Isoperimetrical conditions are formed with the help of the *"Effectiveness Function"* and *"Normalizing Condition"*. The *"Effectiveness Function"* each time is expressed through the so-called *"Cognitive Function"*.

Formulate the two principal variational problems.

Problem *"A"*: the *"Subjective Entropy"* is fixed at the specified level; and the *"Effectiveness Function"* is maximized. Since the *"Subjective Entropy"* characterize the degree of inequality, then it is possible to think that in this case the problem the *"Utilitarism"* is being solved.

Problem *"B"*: the value of the *"Effectiveness Function"* is fixed and the *"Subjective Rating Entropy"* is maximized. That is, in a rather general setting, the problem of the *"Egalitarizm"* is being solved.

The authors believe that in such formulation the notions of the *"Utilitarism"* and *"Egalitarizm"* are conditional.

In conclusion of this preamble part we would like to point that the *"Effectiveness Function"* is taken in such view than the principle of "anonymousity" is violated. Some formulas and calculations are presented in the second part. Some important questions were presented in the monograph by one of the authors, namely, entropy thresholds were introduced, entropy areas are not uniform, temperatures were introduced, analogs to some statistical mechanics give us the right to introduce the subjective temperatures, and in the framework of the suppositions many problems of *"Subjective Analysis"* were solved [4-6, 11-16].

The concept of social justice is an object of the decision making theory in a social group [2]. It is considered from the point of view of the "welfare theory" [3].

In accordance to Tokwill (1860) the tendency of a man to an equality is passionate, an eternal, and an irresistible (we can add here — never being realized).

If we say about a State in general, about its features, we can suppose that the State has no other problems besides of elaboration of some determined concept of social justice and to make all possible efforts to realize this concept. If some State has no concept of social justice, it is not a State at all.

Of course such a concept depends on the kind of State. Namely, a slave State has a concept of the social justice which is different from the concept of social justice of a Liberal State.

It is undoubtedly, the social justice is a purely subjective category. The domain of its existence is the brains of the individuals (subjects of the social group). Surely, this concept, as a certain informational object, has its own projection into the domain of the material existence of the socium groups: choice of aims, tasks, strategies, resources and so on.

From the said above it follows that we have the right to apply the methods of the *"Subjective Analysis Theory"* [4-6] to the solution of the whole problem for receiving the numerical estimation.

A concept of the social justice is an ethic category. There are two simplest and oldest concepts: *"Egalitarism"* and *"Utilitarism"*. In between of these marginal cases there are all other compromise cases.

More detailed information about theoretical basement could be found at the site: `http://kasianovv.wixsite.com/entropyofpreferences/thematics`

Theoretical Provisions Development

In the utility theory corresponding criteria were introduced: *"Function of Collective Utility"* [2], (FCU) $W(U)$, where U — utility.

$$U = (u_1, u_2, \ldots, u_M), \tag{1}$$

M — the number of the subjects in the group.

In the case of egalitarism FCU has a form

$$W_e(U) = \min_{j \in \overline{1,M}} u_j. \tag{2}$$

As an additional condition

$$u_{\min} < u_j < u_{\max}, \forall j \tag{3}$$

could be taken into account.

In this case an optimal strategy is the solution of the following extremal problem

$$\mathrm{Str}_{\mathrm{opt}} = \operatorname*{Sup}_{\mathrm{Str} \in S} [W_e(U)] = \operatorname*{Sup}_{\mathrm{Str} \in S} \left(\min_{j \in \overline{1,M}} u_j \right). \tag{4}$$

Egalitarism is a strategy when the poorest member of the group dictates the choice of the strategy of the group.

Egalitarism leads to the leveling of the utilities of all members of the group. But it does not require the full equality between the members. So, egalitarism means that the poorest member is the dictator. By the way, as a result of a revolution the revolutionary expropriation could be realized in accordance with the Pigot-Dalton principle. Such revolution may be named an egalitary one.

In the case of a utilitarism the following function of FCU should be chosen

$$W_u(U) = \sum_{j=1}^{M} u_j, \tag{5}$$

where u_j — individual utility of the group member.

Optimal utilitarian strategy will be found as the solution of the following extremal problem

$$\mathrm{Str}_{\mathrm{opt}} = \operatorname*{Sup}_{\mathrm{Str} \in S} [W_u(U)] = \operatorname*{Sup}_{\mathrm{Str} \in S} \left(\sum_{j=1}^{M} u_j \right). \tag{6}$$

It could be imagined another intermediate cases, for example, when

$$W(U) = \max_{k \in \overline{1,M}} u_k, \tag{7}$$

when

$$U_{\min} < u_k < U_{\max}. \tag{8}$$

In a more general case

$$W(U) = \frac{1}{M} \sum_{j=1}^{M} u_j. \tag{9}$$

Or we could take the FCU function in the form of

$$W(U) = \sum_{j=1}^{M} g_j u_j, \tag{10}$$

where

$$\sum_{j=1}^{M} g_j = 1. \tag{11}$$

Then besides of the FCU function it could be taken into account some additional condition. For instance, inequalities like the following

$$u_k \in \left[u_{k_{\min}}, u_{k_{\max}}\right); k \in \overline{1,M}, \tag{12}$$

or

$$W(U) = \sum_{j=1}^{M} u_j \xi_j, \tag{13}$$

where ξ_j — some ratings

$$\xi_j \in (0 \ldots 1); \left(j \in \overline{1,M}\right). \tag{14}$$

If ξ_j — rating coefficients, then they could be found as a solution of an extremal problem with the functional like Jaynes-problems functions, with the main term, rating entropy:

$$H_{\xi_j} = -\sum_{j=1}^{M} \xi_j \ln \xi_j, \tag{15}$$

which corresponds to the problem of the subjective equality.

We have to say here that the category of the "Social Justice" is an ethical category; and that is why we describe it in the terminology of Subjective Analysis.

If

$$H_{\xi} = H_{\max} = \ln M, \tag{16}$$

it corresponds to the complete (absolute) subjective equality.

Let us see how to imagine the requirements of equality in the terms of Subjective Analysis. Much more precisely, how to connect it with the main principle of Subjective Analysis — the Principle of the Subjective Entropy Maximum.

The entire (complete, absolute) equality could be expressed as an equality of the individual rating coefficient in a group. In this case

$$\xi_j = \frac{1}{M}, \tag{17}$$

for all subjects (each individual) and the entropy then equals

$$H_\xi = \ln M. \tag{18}$$

For all cases of inequality: not every $\xi_j = \dfrac{1}{M}$

$$H_\xi = -\sum_{j=1}^{M} \xi_j \ln \xi_j < \ln M. \tag{19}$$

We could write down the measure of the inequality as a following criterion

$$\boxed{\Phi_0 = \ln M - H_\xi = \ln M + \sum_{j=1}^{M} \xi_j \ln \xi_j.} \tag{20}$$

One of the very important parts of the social justice is the set of the requirements of the safety of each individual, so as the safety of the whole group.

The requirements of the safety could be expressed in the terms of the conflict theory. This theory has been developed in the form of subjective analysis [4-6]. In this theory any conflict is identified as a connection between distributions of preferences. In this case we should distinguish some different kinds of conflicts.

- Inner conflict — conflict between two preferences distributions produced by the same individual consciousness (self-conflict).
- Interpersonal conflict — conflict between distributions of preferences of the same kind, produced by different individuals on the same set of alternatives.
- Conflict between different groups of subjects (inter-group conflict).
- Cold conflict
- Hot conflict
- Object conflict — conflict between two distributions of object preferences.
- Subject conflict — conflict between two subjects or between two groups of subjects.

The optimization criteria is written in the form of

$$\boxed{\Phi_\xi = H_\xi \pm \beta W + \gamma \sum_{j=1}^{M} \xi_j,} \tag{21}$$

where β and γ — the so-called structure parameters.

More detailed notation is

$$\boxed{\Phi_\xi = -\sum_{j=1}^{M} \xi_j \ln \xi_j \pm \beta \sum_{j=1}^{M} \xi_j u_j + \gamma \sum_{j=1}^{M} \xi_j,} \tag{22}$$

where ξ_j — index of the absolute ratings.

Strictly speaking we have to define the subject, in whose brain this rating distribution is realized. In the opposite case we could not have used the *"Subjective Entropy Maximum Principle"*.

For example, it could be taken that the group contains $n+1$ subjects; one of which $j = k$ is the *"External Observer"*.

If we introduce conditional ratings $\xi(k \to j)$ — the rating of j in the eyes of k, we can receive a simple enough algorithm for solving the whole problem. A more detailed approach is presented in the monographs of [5, 6].

In this simple approach $u(j \to k)$ expresses the social justice interests for the entire socium. The theory gets certainty.

In [5, 6], the so-called *"Mutual Utility Theory"* has been developed. It can be used for further development of the given problem.

The situation is illustrated in Figure 1.

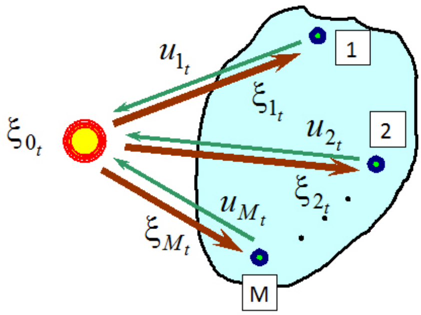


Fig. 1. — Relations of the *"External Observer"* with the evaluated group

Here we can use a supposition about existence of the thresholds of the characteristics mentioned above [6].

Several thresholds were introduced in [5]. First of all such thresholds define the levels of the entropy of a decision making — the choice of a strategy or alternatives. It is designated as H^*.

It means that two conditions are fulfilled:

1) $H_\xi \leq H^*$ at the time t^*.

2) $\dfrac{dH_\xi}{dt} < 0$ at the moment of t^*.

Secondly, it is supposed that another threshold H_{**} defines the level, down of which there is an area of the ratings utilitarism (dictator's regime).

At last there exist such a level of entropy that the only alternative seems to be available, and distribution of preferences becomes singular. In this case the state (condition) of psych could be named *"Zombie Level"*. It means by the way that there are no resources in the system in order to drive the psych of the subject out from this state.

Exceeding the other threshold level H_* puts the psych down into a hysteria if $H \geq H_*$.

This is portrayed in stripes in Fig. 2.

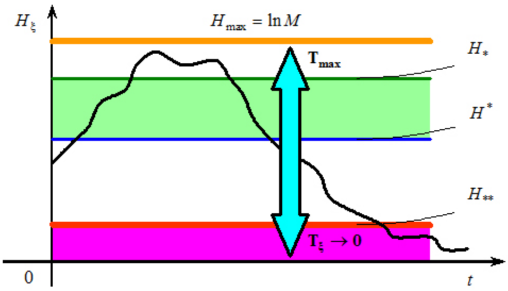


Fig. 2. — Structure of the subjective entropy space

So, the domain (realm) of $[H_{\max}, H_*]$ is the domain of the psych stress state. The domain $(H_*, H^*]$ — realm of freedom, domain $(H^*, H_{**}]$ — realm of necessity, and at last the domain $(H_{**}, 0]$ — *"Zombie Domain"*. Let us repeat once again, that there are no possibilities to get individuals out of this domain.

In Fig. 2 it is shown the structure of the subjective entropy space and mutual positions of the corresponding thresholds. These positions depend upon the social temperatures.

In what way can we introduce the social temperature: $T_\xi = \beta_\xi^{-1}$?

The principle of the subjective entropy maximum gives the following distribution of the rating preferences:

$$\xi(j) = \frac{e^{-\beta_\xi U_j}}{\sum_{q=1}^{M} e^{-\beta_\xi U_q}}. \tag{23}$$

Here U_j — is the utility of the subject j. $\xi(j)$ — the integral rating of the subject. β_ξ — could be called the inverse social temperature.

The entropy of the subjective rating preferences is equal

$$H_\xi = -\sum_{j=1}^{M} \xi(j) \ln \xi(j). \tag{24}$$

Description (23) formally coincides with the Gibbs description in kinetodynamics, where β is the inverse temperature.

"Social Temperature" makes a big influence upon the social system behavior mentioned above.

If T_ξ tends to infinity, then the state of psych tends to hysteria state. If, on the contrary, T_ξ tends to "0", then the state of psych tends to the "Zombie State".

In the general sense of the "Social Justice Category", the requirement of safety of the whole system and separate individuals should be put as the one of the main requirements.

Because the conflicts of the different kinds are the sense of social systems existence, then the main requirements of the social safety could be expressed more strictly.

Let us apply, previously introduced and used for conflict sharpness evaluation in work [11], some criteria of social justice and safety in the view of the "Conflict Tension" between two subjects $K_{1,2}$, which is a function of the two rating entropies: H_{ξ_1}, H_{ξ_2} and the coefficient of correlation ρ_{ξ_1,ξ_2} [4] between the rating distributions ξ_1 and ξ_2 in the group:

$$K_{1,2} = f\left(H_{\xi_1}, H_{\xi_2}, \rho_{\xi_1,\xi_2}\right). \tag{25}$$

Of course, some other criteria can be proposed. Nevertheless any of them should have a certain interpretation in subjective analysis terms.

In accordance with the statements told above some kind of conflicts could be introduced [6]. We accept that in the well organized social systems the administrative ranks are everywhere growing functions of their arguments of the social ratings. In such situations the opportunities for internal social conflicts are low. Otherwise the negative correlation between the ranks and ratings distributions could be a source of a social conflict.

There are a lot of possibilities for the measures of such ranks. For example, consider the following functional originating the recursion scheme:

$$\Phi_{\xi_{t,t+1}} = -\sum_{j=1}^{M} \xi_{j_{t+1}} \ln \xi_{j_{t+1}} \pm \beta_{\xi_t} \sum_{j=1}^{M} \xi_{j_{t+1}} u_{j_t} \pm \alpha_{\xi_t} \sum_{j=1}^{M} \xi_{j_{t+1}} \ln \xi_{j_t} + \gamma_{\xi_t} \sum_{j=1}^{M} \xi_{j_{t+1}}, \tag{26}$$

where β_{ξ_t} and α_{ξ_t} — corresponding structure parameters.

From condition

$$\frac{\partial \Phi_{\xi_{t,t+1}}}{\partial \xi_{j_{t+1}}} = 0, \left(j \in \overline{1,M}\right), \tag{27}$$

the rating distribution is being found:

$$\begin{aligned}
-\ln \xi_{j_{t+1}} - 1 \pm \beta_{\xi_t} u_{j_t} \pm \alpha_{\xi_t} \ln \xi_{j_t} + \gamma_{\xi_t} &= 0, \\
\ln \xi_{j_{t+1}} &= \gamma_{\xi_t} - 1 \pm \beta_{\xi_t} u_{j_t} \pm \alpha_{\xi_t} \ln \xi_{j_t}, \\
\ln \xi_{j_{t+1}} &= \gamma_{\xi_t} - 1 \pm \beta_{\xi_t} u_{j_t} + \ln \left(\xi_{j_t}\right)^{\pm \alpha_{\xi_t}}, \\
\xi_{j_{t+1}} &= \exp\left[\gamma_{\xi_t} - 1 \pm \beta_{\xi_t} u_{j_t} + \ln \left(\xi_{j_t}\right)^{\pm \alpha_{\xi_t}}\right] \\
&= \exp\left[\gamma_{\xi_t} - 1\right] \cdot \exp\left[\pm \beta_{\xi_t} u_{j_t} + \ln \left(\xi_{j_t}\right)^{\pm \alpha_{\xi_t}}\right], \\
\sum_{j=1}^{M} \xi_{j_{t+1}} = 1 &= \exp\left[\gamma_{\xi_t} - 1\right] \cdot \left\{\sum_{j=1}^{M} \left[\left(\xi_{j_t}\right)^{\pm \alpha_{\xi_t}}\right] \cdot \exp\left[\pm \beta_{\xi_t} u_{j_t}\right]\right\}, \\
\exp\left[\gamma_{\xi_t} - 1\right] &= \frac{1}{\sum_{j=1}^{M} \left[\left(\xi_{j_t}\right)^{\pm \alpha_{\xi_t}}\right] \cdot \exp\left[\pm \beta_{\xi_t} u_{j_t}\right]}, \\
\xi_{j_{t+1}} &= \frac{\left[\left(\xi_{j_t}\right)^{\pm \alpha_{\xi_t}}\right] \cdot \exp\left[\pm \beta_{\xi_t} u_{j_t}\right]}{\sum_{q=1}^{M} \left[\left(\xi_{q_t}\right)^{\pm \alpha_{\xi_t}}\right] \cdot \exp\left[\pm \beta_{\xi_t} u_{q_t}\right]}.
\end{aligned} \tag{28}$$

Numerical Simulation

The recursive scheme model (26)–(28) for the simplest case is realized with a three subject set system.

The computational initial data have been accepted as follows: $\xi_{1_0} = 0.32$; $\xi_{2_0} = 0.5$; $\xi_{3_0} = 0.18$. The rest of the values are: $\alpha = \beta = 0.8$; $u_{j_t} = y\xi_{j_t}$; $y = 0.68$.

A more developed case is when $u_{1_0} = 0.6$; $u_{2_0} = 0.70698$; $u_{3_0} = 0.8$;

$$\left.\begin{aligned}
u_{1_{t+1}} &= u_{1_t} \\
u_{2_{t+1}} &= u_{2_t} \\
u_{3_{t+1}} &= u_{3_t} \\
\xi_{1_{t+1}} &= \frac{\left[\left(\xi_{1_t}\right)^{\alpha_{\xi_t}}\right] \cdot \exp\left[\beta_{\xi_t} u_{1_t} \xi_{1_t}\right]}{\sum_{q=1}^{3}\left[\left(\xi_{q_t}\right)^{\alpha_{\xi_t}}\right] \cdot \exp\left[\beta_{\xi_t} u_{q_t} \xi_{q_t}\right]} \\
\xi_{2_{t+1}} &= \frac{\left[\left(\xi_{2_t}\right)^{\alpha_{\xi_t}}\right] \cdot \exp\left[\beta_{\xi_t} u_{2_t} \xi_{2_t}\right]}{\sum_{q=1}^{3}\left[\left(\xi_{q_t}\right)^{\alpha_{\xi_t}}\right] \cdot \exp\left[\beta_{\xi_t} u_{q_t} \xi_{q_t}\right]} \\
\xi_{3_{t+1}} &= \frac{\left[\left(\xi_{3_t}\right)^{\alpha_{\xi_t}}\right] \cdot \exp\left[\beta_{\xi_t} u_{3_t} \xi_{3_t}\right]}{\sum_{q=1}^{3}\left[\left(\xi_{q_t}\right)^{\alpha_{\xi_t}}\right] \cdot \exp\left[\beta_{\xi_t} u_{q_t} \xi_{q_t}\right]}
\end{aligned}\right\} . \tag{29}$$

The results are presented in Fig. 3.

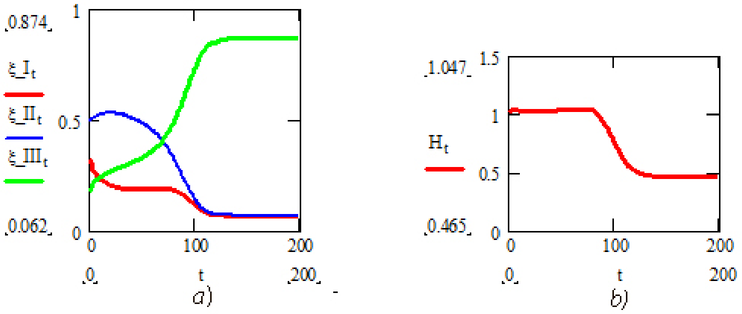


Fig. 3. — Ratings *a)* and entropy *b)*

For the social justice measures it is proposed to use the coefficients of

$$\mathrm{K}_{1_t} = \rho_{\Sigma_t}\left(1 - \overline{H}_{j_t}\right)^{\delta}\left(1 - \overline{H}_{k_t}\right)^{\delta} \tag{30}$$

and

$$\mathrm{K}_{2_t} = \rho_{\Sigma_t} - \mathrm{K}_{1_t}, \tag{31}$$

where the coefficient of correlation ρ_{Σ_t} with respect to the alternative utilities is calculated by such formula

$$\rho_{\Sigma_t}(j,\, k) = \frac{\sum_{i=1}^{N}\left(\xi_t\left(j\,|u_i\right) - \dfrac{1}{N}\right)\left(\xi_t\left(k\,|u_i\right) - \dfrac{1}{N}\right)}{\sqrt{\sum_{i=1}^{N}\left(\xi_t\left(j\,|u_i\right) - \dfrac{1}{N}\right)^2 \sum_{i=1}^{N}\left(\xi_t\left(k\,|u_i\right) - \dfrac{1}{N}\right)^2}}, \tag{32}$$

and the relative entropies $\overline{H}_{j_t} = \dfrac{H_{j_t}}{H_{\max}}$ and $\overline{H}_{k_t} = \dfrac{H_{k_t}}{H_{\max}}$, and δ — sensitivity index.

The values of (30) — (32) are explained in a sufficient form in work [11].
The social justice modeling results, in terms of (30) — (32) for the subjects of 1 and 2 at $\delta = 0.1$, are shown in Fig. 4.

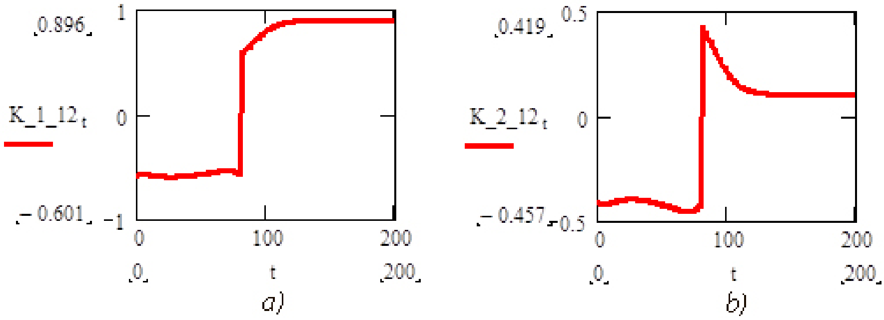


Fig. 4. — Social justice measures *a)* and *b)*

The same experiment results for the subjects of 1 and 3, as well as for 2 and 3 are illustrated in Fig. 5 correspondingly.

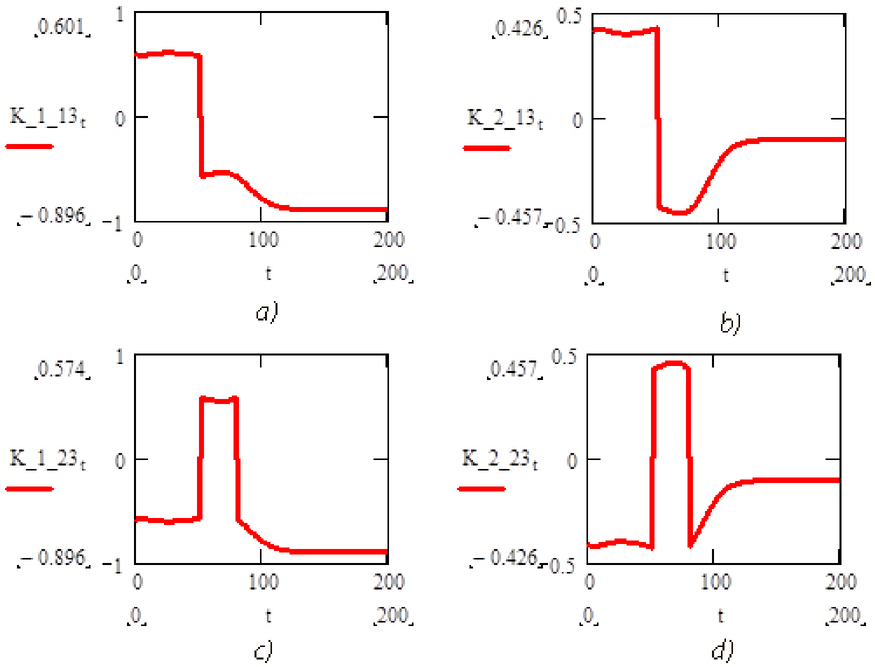


Fig. 5. — Social justice measures *a)–d)*

Next up is the case when changing parameters of the model it is possible to forecast the development of situations in the group of three subjects, for example, with three political parties.

$$\left.\begin{aligned}
u_{1_{t+1}} &= u_{1_t}\left[1+\left(1-\frac{H_t}{H_*}\right)\left(\frac{H_t}{H_{**}}-1\right)\xi_{1_t}\right]\\
u_{2_{t+1}} &= u_{2_t}\left[1+\left(1-\frac{H_t}{H_*}\right)\left(\frac{H_t}{H_{**}}-1\right)\xi_{2_t}\right]\\
u_{3_{t+1}} &= u_{3_t}\left[1+\left(1-\frac{H_t}{H_*}\right)\left(\frac{H_t}{H_{**}}-1\right)\xi_{3_t}\right]\\
\xi_{1_{t+1}} &= \frac{\left[(\xi_{1_t})^{\alpha_{\xi_t}}\right]\cdot\exp\left[\beta_{\xi_t}u_{1_t}\xi_{1_t}\right]}{\sum_{q=1}^{3}\left[(\xi_{q_t})^{\alpha_{\xi_t}}\right]\cdot\exp\left[\beta_{\xi_t}u_{q_t}\xi_{q_t}\right]}\\
\xi_{2_{t+1}} &= \frac{\left[(\xi_{2_t})^{\alpha_{\xi_t}}\right]\cdot\exp\left[\beta_{\xi_t}u_{2_t}\xi_{2_t}\right]}{\sum_{q=1}^{3}\left[(\xi_{q_t})^{\alpha_{\xi_t}}\right]\cdot\exp\left[\beta_{\xi_t}u_{q_t}\xi_{q_t}\right]}\\
\xi_{3_{t+1}} &= \frac{\left[(\xi_{3_t})^{\alpha_{\xi_t}}\right]\cdot\exp\left[\beta_{\xi_t}u_{3_t}\xi_{3_t}\right]}{\sum_{q=1}^{3}\left[(\xi_{q_t})^{\alpha_{\xi_t}}\right]\cdot\exp\left[\beta_{\xi_t}u_{q_t}\xi_{q_t}\right]}
\end{aligned}\right\}, \tag{33}$$

where H_* and H_{**} — entropy marginal values which symbolize the utilities' property to maximize at some intermediate value of the ratings entropy.

Such idea is based upon the mathematical model implying the illustrated in Fig. 6 effect with $H_* = (x = 0.9)$ and $H_{**} = (x = 0.1)$.

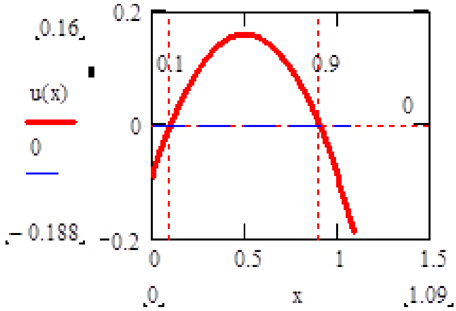
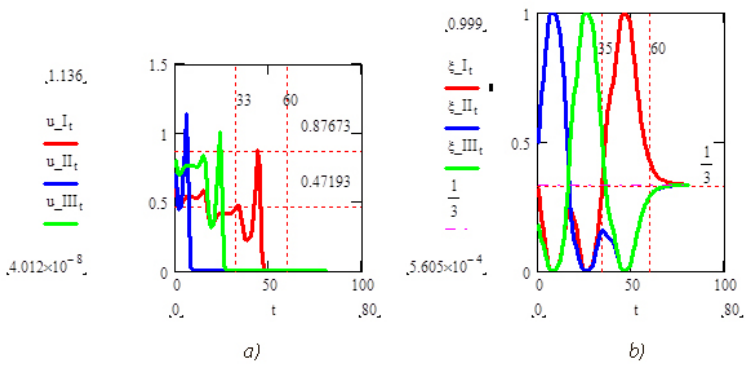


Fig. 6. — Utilities-social ratings entropy dependence concept

In the presented paper experimentations $\beta = 3$; $\alpha = 0.8$; $H_* = 0.8\ln 3$; $H_{**} = 0.2\ln 3$.

The computer simulation results for the subjects of 1–3 are shown in Fig. 7 correspondingly.



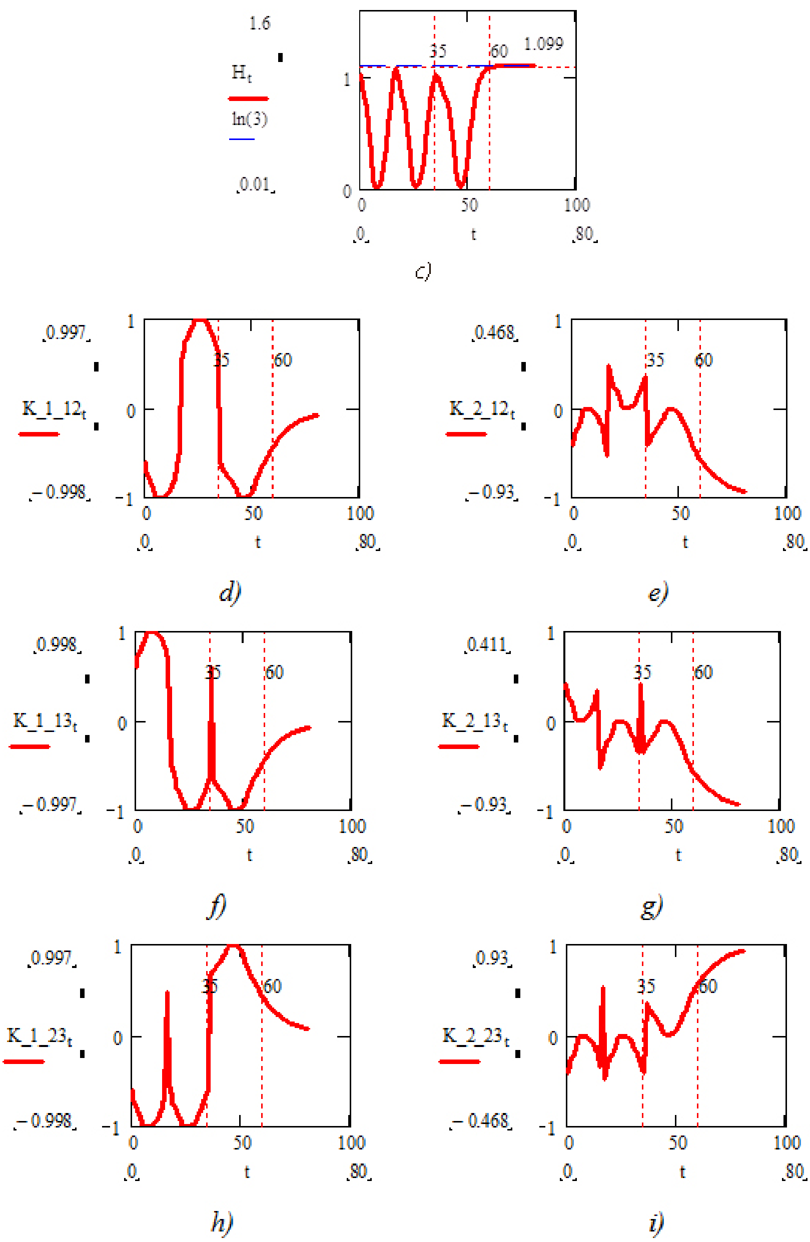


Fig. 7. — Utilities *a)*, ratings *b)*, entropy *c)*, and social justice measures *d)–i)*

Diagrams in Fig. 7 show the possible developments of social or political and economical situations with respect to the justice measures within a long enough period of time.

Analysis of Results and Discussion

As that follows from the plotted in Fig. 1–7 illustrations and diagrams, the subjective understanding of the social justice idea is grounded upon the accepted by the socium participants (individuals, human beings) imaginations about personal utilities, individuals' social ranks (ratings), with the significant impact of the ratings uncertainty measure. In the proposed herewith paper, these notions form the dynamical recursive systems, likewise (28), (29) or (33).

The ratings entropy, being a measure of the ratings uncertainty, in conjunction with the ratings themselves also influences the utilities. The recursive procedure of the mathematical model system similar to (28), (29) or (33) allows simulations of the social dynamics phenomena widely described and discussed in the trendy literature of this classification [1-6].

For instance, the effects of the egalitarian, pluralism, society might lead to the anarchy chaos visible in Fig. 7 *c)* entropy diagram (the ratings entropy tends to its climax value of ln3; the crucial point here is with the coordinates of $t = 60$ and $H_t = 1.0839$); when the ratings of the social groups (political parties, communities) converge (also see Fig. 7 *b)* curves after 60 iteration steps, let us say conditional time units, years for long term perspective prognostic simulations).

Thus, political parties' ratings become equal; the situation is of the complete uncertainty; social indifference; neglecting citizens rights or voters duties etc, resulting in tending to zero utilities (see Fig. 7 *a)* curves as well).

Remarkable here is the fact that the memory effect has its impact. The local extremum of the first party utility of $u_{1t} = 0.47193$ at $t = 33$ (see Fig. 7 a) red curve) results in the party's # I maximal rate (see Fig. 7 b) red curve's maximal positive differential).

Election cycles attached to the specific time intervals might be modeled in that style.

The similar features are noticeable in Fig. 3–5 too.

Conclusions

Thus, in the paper, some simple model of the social group behavior dynamics has been proposed. The main peculiarity of the model is the implementation of the new psychological principle of the maximum subjective entropy. This gives a possibility to consider the psychological problems and investigate the influence of psychology upon the social justice phenomenon. The theory has been applied to the issues of social behavior, being investigated as a dynamical process of justice. That allows receiving the numerical estimations.

It is for sure, that such approach, to the problem solution, could be useful to forecast some social events developments connected with the problem of social justice.

It is seen that further problem is the problem of identification and structure parameters of the model. To make the results more realistic. Besides, farther development of the theory may be based upon preferences of the first kind — objective preferences.

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